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ABSTRACT

To gather the information for this section, visits were made to each of ten major curriculum development projects. These were selected because of the impact which each has had on curriculum reform--past, present, and future. It was not possible to visit every project, and several of the more significant ones, such as the Greater Cleveland Mathematics Program, the Nuffield Project, and Patterns in Arithmetic, are not included in this section. Materials on the University of Illinois Arithmetic Project also are not included. The record of a press conference with Jean Piaget and Barbel Inhelder are, however, appended. Pertinent information on each project was briefly summarized, and notes made during the visit included as a preface to the taped interview with the director of each project. In this interview, his viewpoints about a dozen significant questions are recorded.

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INTERPRETIVE STUDY OF RESEARCH AND DEVELOPMENT IN ELEMENTARY SCHOOL MATHEMATICS

VOLUME 3:
DEVELOPMENTAL PROJECTS

Marilyn N. Suydam
C. Alan Riedesel

The Pennsylvania State University
University Park, Pennsylvania 16802

June 30, 1969

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OVERVIEW

This Final Report of Phase I of the Interpretive Study of Research and Development in Elementary School Mathematics is bound in three volumes. Volume 1 describes the study and presents the summarized findings, in a form which should prove useful to teachers and principals. Volume 2, containing the compilation of categorized research reports, will possibly prove to be primarily of use to researchers. In Volume 3, reports of developmental projects are summarized; those teaching mathematics education courses may find these particularly helpful.

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VOLUME 3

PART IV: DEVELOPMENTAL PROJECTS

Background and Taped Interviews with Directors of Some Current Developmental Projects

To gather the information for this section, visits were made to each of ten major curriculum development projects. These were selected because of the impact which each has had on curriculum reform--past, present, and future. It was not possible to visit every project, and several of the more significant ones, such as the Greater Cleveland Mathematics Program, the Nuffield Project, and Patterns in Arithmetic, are not included in this section. Materials on the University of Illinois Arithmetic Project also are not included. The record of a press conference with Jean Piaget and Barbel Inhelder are, however, appended.

Pertinent information on each project was briefly summarized, and notes made during the visit included as a preface to the taped interview with the director of each project. In this interview, his viewpoints about a dozen significant questions are recorded.

Project Title: Mathematics Project of Sherbrooke
(Centre de Recherches en Psycho-Mathematiques)

Location: University of Sherbrooke, Sherbrooke, Quebec, Canada

Director: Zoltan P. Dienes

Background

One of the most interesting aspects of the program is the work with children in the schools. One can visit a classroom at any level and see children engaged in small group laboratory activities. The teacher acts as a resource and an organizer of the activities. In any one classroom, materials such as multi-base blocks, attribute blocks, logic houses, and rods, in addition to paper-and-pencil activities, are in use. Seldom, if ever, does the teacher address the class as an entity.

The curriculum is certainly of a non-standard type. There is a much greater emphasis on logic, geometric transformations, and non-decimal numeration systems than would be found in a typical modern program. The emphasis is on mathematics, not on arithmetic, with the belief that the arithmetic topics can be incidentally learned through a variety of mathematical experiences. Certainly this contention is open to debate, but some empirical evidence should be collected.

Professor Dienes has conducted projects and workshops related to this type of program in several countries.

INTERVIEW with ZOLTAN P. DIENES, Director
CENTRE DE RECHERCHES EN PSYCHO-MATHEMATIQUES, PROJET MATHEMATIQUE DE SHERBROOKE

C. Alan Riedesel
(20 November 1968)

Q: *What would you say the purposes and objectives of your project over the next few years would be?*

A: Well, it is in two different sections. One is the psychological section and the other is the pedagogical application of the psychological work. The pedagogical work also has two sections, one methodological and the other curricular. So there are really three aspects of the work to consider:

(i) The psychological problems of learning mathematics as from the standpoint of experimental psychology.
(ii) The methodological and the consequent pedagogical problems, as you meet them in the real classroom.
(iii) The curricular problems.

Q: *You are working on a program and materials for the first six grades at the present time. When do you plan to move on into the junior high program?*

A: Well, we have a fairly large number of sixth grade children this year. They will become seventh graders next year and after that they will enter the first year of high school. As the children go through the higher grades, we shall work on the program for these grades. It will probably take us between five to ten years to establish a high school program. It has taken me about ten to twelve years to even begin to establish an elementary school program.

(Interview with Z. P. Dienes -- C. Alan Riedesel)

Q: *What would you consider your philosophical basis to a teaching approach--your guiding principles, if you want, in terms of teaching children?*

A: Well, I think perhaps the chief one would be that I think a child learns most satisfactorily from direct interaction with his environment. If this is indeed so, then the job of the pedagogue is to manipulate the child's environment. So it seems to me that what we have to do is to emerse the child in a mathematical environment if we want him to learn mathematics. We should emerse him in a logical environment if we want him to learn logic, and so on. The result of our efforts will be interaction between the actual way in which his environment is set up and himself. Naturally the environment also includes other children and the teacher. There is interaction between one child and another child; there is intellectual interaction as well as social and emotional interaction. These form important ingredients of my methodology. Possibly the least important of these interactions is the direct one between teacher and child. This must not be misunderstood. I do not mean that the teacher is going to be any less important but that the teacher will play a very much more subtle part in setting up the situation. She must set up the group, the members of which interact among themselves as well as with a specially prepared mathematical environment. In setting up such an environment within which the children can interact, the teacher acts as a kind of peripatetic adviser. She must see that everything is working. She might in a sense be likened to the person who keeps an eye on a big power station. Everything might be ticking over fine, but if things threaten to go a little wrong, he must adjust a gauge here and there. This does not mean to say that this person is not responsible for the entire power station. He has got to see that everything is going correctly and he has got to have it well planned.

In the same way the teacher is fully responsible for what goes on in the classroom. Many people, when they come into one of my chaotic looking classrooms, let out a big gasp and then politely ask, 'what is the teacher doing?' Half the time he might not even be there. He could in fact leave the classroom for say, a quarter of an hour, and the children might not even notice it.

Q: *A lot of literature talks about discovery. How would you approach or orient your teaching?*

A: Well, of course it all depends on what you mean by discovery. If you were to put a lot of rocks in the classroom and a lot of children and say, 'discover mathematics,' it is obvious that not much that is even remotely mathematical will happen. If you put not just rocks but certain particular kinds of materials, then children are more likely to discover mathematics. In addition, from time to time you might make certain suggestions to the children in response to what they have done with the materials. Of course, you are really telling them to discover more mathematics. When and how a discovery happens, is largely up to the activity and the guidance of the teacher and how he or she manipulates the environment, including the social interactions in the classroom.

Q: *What ways do you consider individual differences in the classroom in terms of abilities of children?*

A: Well, you have probably seen that in every classroom we have children working in small groups. Now, some teachers prefer to make reasonably homogeneous groups, but most teachers find that a heterogeneous group is better as long as it is not too heterogeneous. In other words, you cannot put an Einstein with a moron but you can have children fairly different from each other in ability and achievement and still get exciting interaction between the members. So, in this sense, I take note of the individual differences.

Q: *In other words, do you prefer a heterogeneous fourth grade class rather than five sections of fourth grade in a given school having ability groups?*

A: Yes, I would just order them alphabetically so that there would not be a bright class, a middle class, and a dull class.

Q: *Could you very briefly give the major type of objective that you like a sixth grader to have reached when he finishes your program?*

A: A sixth grader. Well, that is a little difficult because you see, by the time children reach the age of eleven, which is presumably sixth grade, there is an enormous spread in their achievement. I wonder if you would like me to say what the least of these is likely to be or what the most might turn out to be? It does not make much sense to talk about an average because the spread is so great. I would say that the average sixth grader would know the normal processes of arithmetic for a start. He would know how to multiply and divide, add and subtract natural numbers and he would be able to do this in a multiple digit situation. He would also know what fractions are, both as states and as operators. He would know how to use fractions in, say, scale drawings, proportions and percentages, and so on. I think all children probably reach this stage.

Apart from the above, most of our sixth graders have a certain minimum knowledge of logic. They know for example some of the elementary properties of implications and equivalences. They also know some of the elementary reasoning patterns that occur in mathematics such as the use of contrapositives. I think most children can learn this much.

As for relations, our average hypothetical sixth grader will have come across equivalence relations, order relations, and difference relations in many different contexts, possibly in geometry, in arithmetic and in algebra.

In geometry they would be familiar with elementary topological notions as well as with the isometries of the easy figures, such as the equilateral triangle and the square if not the regular tetrahedron.

This would be a kind of common core. There would be other geometrical ideas developed, such as incidence, properties of points, lines and planes. This would come in under relations. A point lies on a line. So, "lies on", would be a relation that would be treated under the heading, 'relations'.

What the 'good performer' would or should know is anybody's guess. We have had children learn most of the groups up to the twenty-four group of the rotations of the cube and all its subgroups. They have learned all sorts of transformations between these groups. They have learned the isomorphisms between permutation groups and the rotation and symmetry groups. They have learned about direct products. They have also learned about the axiomatization process for finite systems, as well as for infinite systems such as conventional algebra and so on. So there is no obvious limit to what children can learn. We have even had children doing completeness proofs and incompleteness proofs, of axiom systems, scalar products, matrix multiplications, complex numbers etc.

Q: *What type of activities do you think are necessary for in-service teacher training for the teachers involved in this project?*

A: (1) Elementary school teachers should be involved with children, because most elementary school teachers are in the job because they want to be, not because it is an easy job or because they think it is an easy job, but because they are motivated by the children. So, the way to get at the Achilles heel of the teacher is through the children. They will "buy" things that are "good for the children". So, I would always use demonstration classes using the various kinds of mathematics you are trying to teach them. In this way the teachers will be motivated to learn mathematics.

(2) We need to do more than motivate them, we need to convince them that the children can do it. If you give teachers a lecture, they will say, "No, the children will never understand that." So you have to show them that the children can in fact do some of the things which we advocate. The proof of the pudding is in the eating.

(3) The next ingredient is that teachers should be exposed to a lot of concrete manipulation themselves; in other words they should meet the mathematical environment in exactly the same way as the child is going to meet this environment. So they should experience this interaction with the environment and have discussions among themselves, correcting each other's mistakes and so on.

(4) They should also learn to construct materials in given situations and in the process become more ingenious.

(5) Teachers should also learn how to make up a symbol system so that they can supervise children who are making up symbol systems.

(6) Conversely, they should also learn how to read mathematics. This is a decoding exercise and it is distinct from reading in your native language because there is no redundancy in mathematical symbolism, and loss of attention for a fraction of a second will result in complete lack of comprehension. Ordinary language is very redundant. We can "afford" to miss quite a lot and still "get the message".

To sum up: mathematical reading sessions, mathematical writing sessions, symbolization sessions of mathematics which you have already learned, workshops of learning the mathematics from the environment and many learning situations with children, these are the main ingredients of a good teacher training program. You should have as few lectures as possible.

Q: *By implication, I would say that a lot of the mathematics is perhaps over the head of the typical elementary teacher in the United States --*

some of the topological ideas, some of the ideas in logic. You would prefer to have her learn this through working in groups and working with children as opposed to attending college mathematics courses.

A: Yes. In fact, I would go as far as to say that very few teachers have ever learned any mathematics up to now. That is, I would say that if you took an average school teacher anywhere in the world the amount of mathematics he or she knew would be minimal. And so I think what we really are saying is that teachers now must learn mathematics whereas before they did not.

Q: *You picture that the way to do it is through a combination of teaching and learning mathematics as opposed to just a pure mathematics focus?*

A: I do not think a teacher who is not a born mathematician is able to be lectured at off the blackboard and learn it through a symbolic method any more than the average child is able to. If they were, there would be more people knowing and understanding and enjoying mathematics today.

Q: *How do you picture over a period of time -- I know it is too early for this now -- to evaluate your project? I know research is one of your emphases.*

A: Yes, well -- would you like to enlarge on what you mean by evaluating?

Q: *Oh, I suppose be able to show that it is an effective curriculum for youngsters, be able to show that certain techniques of the material work better than other techniques, and so on. I am not really talking about the comparison with your curriculum and the typical curriculum perhaps but --*

A: You could not do that obviously as our curriculum is radically different from the "normal" ones, whether traditional or "new math". If what you want to know is whether children put through this kind of program are able to reach the same degree of computational facility and the same degree of problem solving facility as other more traditional programs are able to get them to, I think I can say the answer is yes if only because every time such testing has been tried, our children have always achieved at least as good or better results than normal. It seems to me, however, that such a procedure is not evaluating a program. I think evaluating a program means seeing whether the aims which are set forth by the originators of the program are in fact reached and the aims for me are not computational skill and conventional problem solving ability but ability to understand and handle really complex and abstract mathematical situations. Now, I have yet to find ways in which you can test mathematical abstracting ability but we are working on this problem in our "psychological section".

Q: *So, you would perceive the future development and validation of tests getting at these particular traits?*

A: Yes, I am going to try. But let us not fool ourselves. How do you validate such a test? Say I have developed a test; I want to know whether it does, in fact, test for an understanding by a child. How do I do it? Probably I talk to the child and see if he really understands. In other words, I make judgements or somebody makes judgements. In the last resort, any test is validated as against somebody's personal judgement. At the moment we have reached a stage where we can make reasonable judgements of this kind. That is, when we go into one of our own classrooms and talk to the children, we can see immediately to what extent these children understand and are able to handle these mathematical abstractions. To turn such an informal "testing" into a mass testing apparatus is a big problem. It may be quite insolvable. But even if it were, I do not think it would matter, because you know I fail to see how you would do better than making a judgement.

It is how you validate a test in the last resort in any case. Have you ever asked your wife if she had her dress "validated" before she bought it? If people like our program, they use it. Of course, most people do not even know that it exists, much less how we apply it to real children.

Q: *What do you picture as being the great major impact impact of your project and what do you consider the future impact of the project to be on elementary mathematics education, primarily, throughout the world?*

A: Do you mean where?

Q: *No, not as much -- just what type of impact do you picture more than where.*

A: Well, probably I think it is a sense of liberation for both the child and the teacher. You see, knowledge and competence liberate. Now, if you really know mathematics, you know it right down to its foundation, then you are not just symbol-pushing within a symbolic superstructure, but you have got it in your bones. You are confident, you feel free, you do not feel tied down and constrained to go along a certain path where sombody is telling you to go. You saw those children this morning; they seemed happy, didn't they? I do not think there was an unhappy child in any of those classrooms.

Q: *No, I think they enjoyed it.*

A: They all enjoyed themselves. I think the teachers did. So I think this would be a very important contribution, to turn mathematics into a liberating force rather than into a shackle which it very largely is today. I think that is probably the largest impact that I would like to make.

Q: *Do you see any potential dangers of improper use or interpretation of materials from your project or ideas from your project -- are there any safeguards that you would like to suggest to someone beginning to use some of your ideas or some of the material?*

A: Well, I am not God Almighty to tell people how to use material. On the other hand, there are certain dangers, as you say. For instance, one danger is to restrict one's material to one or a few. I have always, for the last twelve years, been trying to advocate the use of many materials, the use of many situations, to stop the child and the teacher from getting bogged down to one situation and then expressing that situation in a language in which case the language expresses that situation and not a mathematical abstraction.

There is some evidence that if only a single material is used, then a transfer to other similarly structured problems is made more difficult than if other materials are used. I have just been reading a report from Trinidad where they did an experiment on the use of Cuisenaire rods. Well, it seems that the Cuisenaire children were able to solve the problems less well than traditionally brought up children and yet they were able to compute more effectively than the traditionally brought up children. Why did this happen? I would suggest that the reason why the Cuisenaire rod children computed better is because they were using a calculating machine which worked very effectively. How you put the rods together, even if you do it in your head, is a very effective associative type of machine and this produces correct results for the computations. But when it comes to transferring the rod situation to some verbally expressed situation, the Cuisenaire children find it more difficult than those who are not tied down to that one material. In other words, if you are going to be restrictive, it is probably better to stay traditional in certain ways. Maybe if they had taken some other tests they would have come out differently. But you know I seem to have heard a lot of these kinds of results, not just through Cuisenaire but also through using multibase blocks for example. It is the same thing. If you just use multibase blocks and nothing else, you are going to get restricted and your transfer ability is going to suffer. This definitely is a danger. There are, of course, many others such as premature symbolization, excessive anxiety "to get on to serious things", the slavish use of the task-card system, etc.

Q: *What are your major plans for the future?*

A: Well, I would like to expand the so called pure research, the psychological research. You have seen some of the machinery we are constructing. It would be nice to arrive one day to at least an approximation to a "black box theory" where you could feed things in and feed things out and be able to predict what kind of learning behaviour is going to be generated under given circumstances. At the moment I know of no psychological theory that could predict what was going to happen under one rather complex situation rather than under some other rather complex situation. So, we are very much in need of research to help us build up a coherent set of facts and co-ordinate these facts into an explanatory and then a predictive theory of mathematical learning behaviour. I would not like to call it learning theory because it is not really learning theory. It is a kind of an account of mathematical learning behaviour or more generally of relational learning behaviour. We should like to advance on this front in the next five or ten years and we are going to try.

Further, I would like to finish the construction of the elementary school curriculum. This is pretty well finished in the sense that we have a tentative solution to the problem. In the first six or seven years we have a good idea of what an average child in an average situation in an average country is capable of assimilating in the way of mathematics. Certain methodological assumptions are made in the assessment of these capabilities. We are naturally still working on this problem.

I would like to underline here that what we suggest here is by no means necessarily a unique solution. For instance, we are suggesting that logic should be taught explicitly. We have no direct evidence to show that if logic were not taught explicitly but something else was taught instead, we would not have as good if not better results. If somebody tomorrow would come along and say, "All right, let us teach recursive functions in first grade and let us not bother about logic and sets, but let us build everything on recursion," he may indeed build a much better mathematics program. And I hope somebody goes and does it. One team of people can only do one thing at a time. So, I am suggesting that here is one program which we have developed and nearly finished, and we would like to consolidate this first. Later we shall go into the problems of constructing alternative programs based on different assumptions.

As "our children" get older, we should like to follow them right up to university level. In this way we hope to construct a high school program along with them. These attempts will no doubt bring a crop of psychological problems. What happens to learning during adolescense which does not happen during earlier childhood? Hopefully, research centers of psycho-mathematics will be formed in other areas so that we are not left so alone in doing this kind of work.

Q: *We have not talked much about the social mathematics of the child outside of the classroom, the type of things he runs into in his environment. Do you feel this is an important facet of mathematics in elementary school or not?*

A: Yes, I think it is extremely important and I would go as far as to say that each teacher should collect problems from the kind of situations in the children's own environment that really occur. Such problems will then have mathematical as well as social relevance to the children concerned. Such problems can be collected by organizing school visits. These should be rather carefully planned and discussed with the children beforehand. Let us say we take the children to the railroad station or factory or farm. The children should be asked questions such as, "Well, what do you think you ought to know about how the railroad is run? What do you think the station master does? What do you think has to be done before anyone goes on a train? How are the schedules arranged?" and so on. Having discussed all the questions we are going to ask, then we make the visit. All the children can take paper and pencil and ask all sorts of questions. Some of these will be of a numerical type, others of a social type. Upon coming back, children can collate all this armory of information about the railroad station. They can set up problems for themselves and for each other about these real situations that they themselves have come across.

Children do in any case go with Mum and Dad to the store. They can be encouraged to bring lists of bargains advertised. Advertisements for posts are also a good starting point. Many children enjoy doing statistical work, collecting information about frequencies and so learning about "what are the chances that .....?" Just because I do not put it in all the books I write, it does not necessarily mean I do not agree with it. I think it is extremely important to have mathematics regarded by the child as something which connects him to life and to his milieu, to his social being.

Q: *I was glad to get that comment, because a lot of your suggestions are not particularly related to that, but yet could be adapted that way. One of the things that you mentioned is developing an integrated curriculum in mathematics, and I think, language and art. Would you comment on that?*

A: Well, this year is our first attempt at doing this. Last year we spent a lot of time discussing how we were going to do it and we decided that one of the integrating elements -- well, the main integrating element -- would be the idea of relation. We find that the idea of relation is fundamental, both in mathematics and in language and in art. We need only to think of relations such as symmetry, contrast, order, insides, outsides, frontiers, relative size, proportions, and so on. All these come into art, geometry, as well as into language. Relations come into language every time we say a sentence in which there is a subject, a verb, and a complement. The verb relates the subject to the complement and so the children learn to use the verb as a relation. When they change the verb they change the relation. When they change the subject and complement, they change the members of the universes which are being connected to each other by the relation expressed by the verb. Since they do this in mathematics, they seem to see no great difference between their language learning and their mathematics learning. One of the first things we do in the first grade is to try to get children to realize how things are related to each other. Before they van relate things to each other, they have to learn about properties of objects. So this is one of our most immediate goals. This may be regarded as language, of course, but it is also mathematics. It is also physics if you like. It is "science" to observe and realize and note what the properties of a certain object are. So we are relating the mathematical work on relations, on sets, on attributes, on the properties of objects, with the language structure. The way you change parts of a sentence is an introduction to different relations between sentences. At the same time it helps children to construct equivalence relations between parts of sentences, and the meaning of syntactical function begins to dawn on them.

Q: *You do this in French. Do you think this would be equally applicable to English or German or another language?*

A: Oh, I think so. English and French are really extremely similar in structure. I really do not think there would be any great difference.

(Interview with Z. P. Dienes -- C. Alan Riedesel)

Q: *In the recent Cambridge report, mention was made concerning the need to integrate science and mathematics. There was quite a bit of talk about this in the United States. What is your feeling about this task of integrating mathematics and science in the elementary school?*

A: Well, you know I do not think one can disagree with this. Obviously science and mathematics must be integrated. It is only a question of terminology: What is mathematics and what is science? I seem to find that quite a lot of the curricular elements that in the United States are termed "science", are simply in our mathematics program. Such elements are properties of objects including geometrical properties and symmetries, sorting and cross-sorting and so on. The latter comes into our logic program. Properties of objects come into our mathematics program and symmetries come into our geometry program. It seems to me that we are already in a sense integrating because we are starting from the real world and trying to get the child to abstract his mathematics from the real world. So we are bound to be teaching observational techniques, we are bound to be teaching sorting. We bring the autumn leaves in from outside and the first thing the children want to do is to sort them out. They sort out the big ones and the little ones and the yellow ones and the red ones and the green ones and so on. And so it seems to me that logic and mathematics and science are automatically related the moment you base your activity on the real world. I do not think you can really avoid this kind of integration. It just happens if you follow the natural dynamics of the child's learning.

Project Title:	African Mathematics Program
Location:	Newton, Massachusetts
Director:	Hugh P. Bradley,
Originators:	J. R. Zacharias, M.I.T. W. T. Martin, M.I.T.

Background*

In 1961, ESI, under the general direction of Jerrold Zacharias, Professor of Physics at Massachusetts Institute of Technology, organized a six-week meeting at which some 40 African and British educators from East and West Africa met with a group of Americans to examine the content of school curricula of many countries of English-speaking Africa.

The Conference made firm proposals that ESI should initiate curricular reform programs for Africa. In particular, programs in mathematics, science, social studies and teacher training were recommended. These proposals were given careful consideration and ESI agreed, initially, to try to act upon a program for the reform of the teaching of mathematics from Standard 1 up to School Certificate. It was felt that the work of curriculum reform in mathematics in the U.S.A. and in Britain was sufficiently advanced to make possible a positive contribution to African education.

Under the direction of Professor W. T. Martin of the Massachusetts Institute of Technology, the program was initiated and, after six months of activity including conferences in Accra and Ibadan with African mathematicians, a mathematics workshop was held at Entebbe, Uganda, in July and August 1962.

The Workshops

This Workshop set the plan for the African Mathematics Program and the pattern of work which has been followed since. There were fifty-four participants representing 13 countries--with 24 participants coming from eleven African countries. It was decided to produce materials using the new approach to teaching mathematics in four areas: text materials for the primary school, beginning with Primary I; text materials for the secondary school, beginning with Secondary I; text materials for use in African teacher training colleges; and tests and examinations based on these materials.

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*This material was prepared by EDI.

The first Workshop produced a full course for Standard I and Form I and a model of a School Certificate test based on modern mathematics, using some objective testing.

In 1963, once again at Entebbe, Uganda, books for Standard II and Form II and for the first year of a training college course were produced. Tests were also prepared for Form I classes using the materials, now called the Entebbe Mathematics Series.

Each year since then the number of textbooks has been increased. By 1968 there is a full series of textbooks on 'new mathematics' for Standard I to Standard VI, and for Form I to the School Certificate year. During July and August, 1967, textbooks for Standard VI were written and the pre-School Certificate Secondary course in Additional Mathematics was completed. At the 1968 Workshop an Entebbe Primary Course Guide and a two-year course for Advanced Mathematics were written. A full set of four textbooks is available for use in teacher training colleges, and a battery of tests covering the work of Standard III, and Form I, II and III, and a model examination for School Certificate based on this new mathematics have been prepared. Since the program started, a total of 61 volumes of textual materials have been published. In these five years some 70 Africans, 22 expatriates, 55 American and British mathematicians and educators have helped to write the mathematical texts.

In 1967 the West African Examination Council set a School Certificate Examination on the work of the Entebbe Mathematics Series, and the Cambridge Overseas Examination Syndicate has promised that when pupils present themselves, a suitable examination will also be offered in East and Central Africa.

In 1965 the Workshop was moved from Entebbe to Mombasa, Kenya, and has been held there each year since that time.

Institutes

It became clear that it was necessary to introduce teachers (45% of whom have completed only primary school) to the new pedagogical approaches written into the texts and to train them to use the new texts. To date, 51 Institutes have been held. Institutes last from ten days to three weeks and are staffed by visiting American mathematicians and mathematics educators and by their African counterparts who have participated in the program. They are organized by the Ministry of Education in the participating country, with EDC's cooperation. Some 2,500 teachers and teacher educators have been introduced to these materials at Institutes. The children in nearly 2,000 classes in Ethiopia, Ghana, Kenya, Liberia, Malawi, Nigeria, Sierra Leone, Tanzania, Uganda and Zambia are being taught this new, or modern, approach to mathematics.

ABC Institute

In addition to the in-country Institutes described above, the ABC Program begun in July 1966, concerns itself with the mathematical education of the senior mathematics tutors of the ten participating countries. Its purpose is to develop in each country a small cadre of people knowledgeable about modern mathematics, capable of making adaptations of the materials to suit local conditions, and ready to undertake, within their own countries, the mathematical re-education of their fellow tutors in smaller colleges, who, in turn, will give suitable training to the teachers in the schools or about to enter the schools. The final task of this project is the preparation of a Sourcebook and audio-visual aids suitable for use in African training colleges and in-service courses.

Status of the Program

It is now clear that some of the early experimental aims of the program have been achieved. All of the participating countries have accepted the idea that 'modern mathematics' must be introduced into their school curricula, although just how this should be done may not be clear. Some of these countries are sufficiently involved to have prepared long-range plans to assure the complete adoption of such mathematics throughout all grades, and some are making plans for local adaptation of the Entebbe materials. The African Mathematics Program is happy to cooperate with local initiative of this kind.

Outside Interest

These African-produced materials have aroused considerable interest in other countries. Britain and the West Indies have already expressed a desire to try the books, and similar interest has been indicated in South America and Pakistan. An American adaptation of the texts is available under a trade-name. Botswana, Lesotho and Swaziland have asked to be supplied with books. The African Education Division of UNESCO has also maintained close contact with this program and the EDC African Primary Science Program--the second target of the 1961 Conference--which was started in 1965.

A further indication of the interest being shown in the material is the decision of Science Research Associates Limited (SRA, Ltd., Newtown, Reading Road, Henley-on-Thames, Oxfordshire, England) to publish commercial editions.

INTERVIEW with HUGH P. BRADLEY, Director
AFRICAN MATHEMATICS PROGRAM

C. Alan Riedesel
(12 December 1968)

Q: *What are the purposes and objectives of the African project, both now and in the next few years?*

A: The program started with the very general purpose to improve mathematical education in Africa. As the Africans saw it, they wanted the new mathematics in the African schools because they thought that by making use of the new movement already existing in the United States and in England, they could leapfrog over some of the problems that had taken place in the developed countries' mathematical education. We tried to create a model program, not necessarily one that all African countries could use but one which they could adapt and develop from. The final purpose was to influence the African ministries of education in adopting modern and up-to-date mathematics and also modern and up-to-date methods of teaching mathematics.

Q: *What's the philosophical basis of the teaching approach? In other words, what type of strategies do you expect the teacher to engage in?*

A: There are certain limitations in the training of the African primary teacher, most of whom have only had about seven to eight years of basic education. We are very strongly in favor of activity methods, children learning through doing. However, despite this, we have taken full account of the formal setup of the average African classroom and the traditional methods of teaching that are habitual in Africa, and we have tried not to be unrealistic in suggesting changes. So I'd say that what actually is happening just now is a small move toward activity teaching without going all the way.

Q: *In what ways do you provide for individual differences in materials?*

A: Within the materials, very little. We have planned college and in-service courses and teachers manuals so that the teachers themselves, knowing the children, will make allowances for those individual differences. But we've prepared only one set of materials which in fact will be used in the classroom situation.

Q: *What are the major objectives that the students should have reached by the sixth grade of the program?*

A: Well, what the Africans would like is to acquire enough knowledge of arithmetic to be able to live in an African society which might develop within the next 20 or 30 years. I think, in addition to this, we're trying to help the African ministers to realize that the mathematical needs of society in the next 20 or 30 years will be more than arithmetic, and there will at least be some knowledge of such things as probability, the use of statistics, score-taking, the use of money, things of that sort. Basically, the primary aim is a social aim as far as the African ministries are concerned and we've tried to conform to their wishes in this, no matter what else we attempt.

Q: *How is your project being or going to be evaluated?*

A: Well, we've had informal evaluations from the beginning by people visiting classrooms, talking to African ministers, African teachers, and African children. We have a testing program which I think measures the depth of understanding of the concepts which we are attempting to put over. As far as I know, there is no formal objective evaluation of any educational practice in Africa at the moment, the reason for this being, of course, that there are no standards against which to measure. In fact, we feel that our tests in the African math program in very many ways may become the measure against which new tests may be set. We ourselves are structuring our new tests on the basis of our earlier experiments because there is nothing else to go on. As far as other evaluation, this again is rather informal: visitors going into African classrooms, speaking to African teachers and African ministers, and saying 'what do you think?' I think the other test which is really not quite so informal is that the Africans themselves are measuring the arithmetical ability at primary level by the leading certificate examinations or yearly examinations. They seem to be satisfied that the new mathematics is certainly at least as good for these children as they handle normal computations as for children using traditional texts. They are also aware that these children are learning many additional things that the traditional courses do not offer.

Q: *Do you have any research evidence on the project?*

A: We have no research evidence in American terms. We have some item analysis of questions and things of that sort. I wouldn't say that we could draw any firm conclusions.

Q: *What do you picture to be the major impact of the project?*

A: The impact of the program, I think all of us feel, is not so much in terms of the classroom at all; but we have, we feel, created almost a revolution in the African attitude toward mathematics. When we started off, the African ministries were certainly not deeply interested in the teaching of mathematics or even aware of the fact that change was taking place or that change was necessary. Now we can say very firmly as of last summer that every African ministry of the countries participating have stated that they now want to have new modern mathematics throughout their school systems. And that, happening over a period of seven years, I think is a tremendous impact.

(Interview with Hugh P. Bradley -- C. Alan Riedesel)

Q: *Do you see any potential dangers of improper use of your interpretation of materials of the project?*

A: I think there is always a danger of materials being poorly taught. I think in measuring those dangers you ought to consider how well the previous material has been taught. I think the traditional materials were being taught in a very, very poor way. If you're going to teach something poorly, you're probably doing as well to teach something good, poorly, than something bad, poorly.

Q: *What are the major future plans of the project?*

A: Well, we are moving very much towards teacher education. The experimental program for developing classroom material for the schools is over. Now we're thinking very much how the teachers throughout all the countries can be given the training to help them use the materials. We hope to tackle this through working with the teacher training colleges and training the tutors of these colleges initially to introduce proper courses into their own colleges, but at the same time to give them materials, help, and advice on running in-service courses for teachers within their own college area.

Q: *How much emphasis do you place on 'everyday' mathematics and social mathematics in such a program?*

A: I think we all feel that at the primary school level you can't ignore this factor and it certainly is the desire of the African ministers that this should be handled very thoroughly. We think we have handled it thoroughly and indeed we feel that we have put in the additional ingredients which the African ministers probably weren't fully aware of.

Q: *Was there any attempt to integrate science and mathematics on this?*

A: I wouldn't say that the entire math program itself has attempted this. I think we'd like to. But the problem in Africa is that when the teachers are so unsure of themselves and their knowledge of substance and content, and they are so unsure of themselves even in teaching ability, to go too quickly into an integration you could in fact be ruining the teacher. I think they've got to be sure of themselves within the various areas before we can really integrate properly. There is an extra difficulty, of course, which applies equally to America and that is the general feeling that math is a structured subject and primary school science certainly doesn't look to be structured. And how you integrate unstructured with structured is something which is very difficult. In America, quite a number of us feel that math need not be structured in grades 1, 2 and 3 and we would like to carry this into Africa. Of course we would be reluctant to do it too quickly in case it causes too many anxieties among the teachers.

Q: *What materials have already been produced?*

A: Let's just say it in general. We've covered a full course for primary school, that would be roughly from grade 1 to grade 6. We've covered a full course for secondary school which takes children from about the age of 12 right through into what would be the equivalent of a first year American university. Plus a full course for African primary training colleges, which we feel is appropriate for both the higher grade and lower grade colleges. We have also prepared suitable examinations at the various levels.

Project Title: Cambridge Conference on School Mathematics (CCSM)
Location: Newton, Massachusetts
Director: Hugh P. Bradley
Originators: W. T. Martin, M.I.T. and Gilbert Oakley, Jr., E.D.C.

Background

The program developed from a 1963 conference of mathematicians who explored curriculum reform needs in mathematics. The report of the conference, *Goals for School Mathematics*, presented the philosophy and orientation for the program, and an outline of the content which might be included in our schools by the year 2000. While the intent of CCSM was not primarily to develop materials for classroom use, it seemed necessary to produce some to demonstrate feasibility. These are being developed with teachers, with plans to develop pre-service programs as well as in-service courses with the teachers-of-teachers at various colleges.

INTERVIEW with HUGH P. BRADLEY, Director
CAMBRIDGE CONFERENCE ON SCHOOL MATHEMATICS

C. Alan Riedesel
(12 December 1968)

Q: *What are the purposes and objectives of the Cambridge Conference, both now and as you see them in the next few years?*

A: The Cambridge Conference is closing down as an operation as of the end of December of this year. So as an organization it has no future. There may be something growing out of the ashes but it won't be called the Cambridge Conference. In the past it was an organization which kept an eye on what was happening to mathematical education of America, to make sure that the individual projects that were operating were not ignoring aspects of mathematics which the people in the Conference felt were important. This meant that it always planned to look beyond the present into the future and in so doing, it attempted to draw attention to what seemed to be useful goals for mathematics in the long run. Consequently, almost immediately, the Conference came to the problem of 'if these goals were to be achieved, what sort of education for teachers would be necessary if they are going to teach the mathematics proposed for the 1980's?' And similarly, in examining the goals we almost were forced into the question of the integration of math and science in the primary school. So in general we've looked at mathematics rather from above the level of creation of materials--on what would be desirable objectives to obtain and possible ways of obtaining those objectives.

Q: *If someone were to define the ideas in the Cambridge Report, what type of teaching would be in keeping with this project?*

A: Very much child activity, very thoroughly child activity. I think we could say very strongly that the idea of making the textbook the key of primary school mathematics was contrary to anything we had been thinking of.

Q: *In what ways would you think individual differences would be planned for if this were being carried out by you?*

A: The Cambridge Conference didn't operate in that area. We rather set general goals and weren't thinking of individual goals. We were thinking these were general goals of society. It's quite obvious that pupils with greater ability will maybe achieve these goals earlier than the others. The goals that were written into the 1963 Report, in fact, are a good example of this. But in the long run we feel that all children should be able to learn the sort of mathematics that has been included in *Goals for School Mathematics*.

Q: *What are the major objectives that the student should have reached when he completes sixth grade in the program? I've heard it stated in magazine articles that the Cambridge Report is for the top ten or fifteen percent of the whole population. Would you agree with that?*

A: No, I disagree entirely. In fact, we feel in our feasibility studies that this is being disproved--that if the ideas in the Cambridge Conference and the sort of activity methods that we're using in the feasibility studies are carried into other classrooms, underachievers in mathematics will be able to learn the mathematics which people say is suitable only for the brighter pupil.

Q: *What type of activities would be necessary for in-service training of teachers who would be involved in teaching this type of mathematics?*

A: Well, this is what we're considering just now. Present thinking, which is still very preliminary, is that the teacher-training course must include mathematics beyond the level at which the primary school teacher is going to have to teach. The teacher must have a deeper understanding of the concepts he's teaching, and also what those concepts lead to. But even more, we think that it's important that the actual methods that are used in training teachers within the college should include the very activity methods which are being encouraged for the children in their classroom. We already have some evidence from studies that the underachievers in training colleges are learning mathematics much, much better as a result of activity approaches in the training colleges.

Q: *Do you have some research evidence that the Cambridge Report type of mathematics is working?*

A: Well, we will have by the end of February reports of five feasibility studies: one from Indiana University, one from Boston State College, one from the University of California at Berkeley, one from a school study at Princeton, and one from Teachers College at Columbia.

Q: *Would this be in terms of training teachers to teach this mathematics for children also?*

A: This would be evidence of teachers receiving the sort of training that we've been recommending. It will be that at least. One of them we suspect has gone back traditionally.

Q: *What do you picture as being the major impact of this project?*

A: We never expected to have any strong impact directly on activities in the schools--we wanted to be able to influence the people who are involved in curriculum development both for pupils and for teacher training. We never thought of ourselves as an activist program in the development of school materials. Our purpose has always been to get those involved in curriculum development to come together with such other people as we thought would be useful, to discuss where we should be going, what we should be doing, and what school mathematics in the United States should be.

(Interview with Hugh P. Bradley -- C. Alan Riedesel)

Q: *Do you see any potential dangers of improper use of mathematics materials such as the type recommended in the Cambridge Report?*

A: Well, I think it would be pretty useless for any teacher to try to teach some of the mathematics that was recommended in the *Goals* if the teacher himself had not been properly trained in that mathematics. I think there is already evidence that even when materials like those by S.M.S.G. are put in the hands of the traditional teacher who has been teaching for twenty years, she does not teach the new material very well. I'm not sure, but in this case I think that maybe her old traditional type of teaching was better teaching than her teaching of the new mathematics. It is rather contrary to what I said is happening in the African Program. It's a different situation.

We think it's important of course that those who are creating the materials should definitely pay attention to what we've planned. But it's quite clear that these other reform programs are doing a very good job despite anything we said. We were hoping merely that our influence would make these programs better. As many of the people who are involved in the reform programs have also been part of the Cambridge Conference, I don't think that the dialogue which took place could do anything but help.

Q: *I think you touched on this, but should, in a program like this, science and mathematics be integrated quite often?*

A: Well, we have a report which is published that is now recommending integration of math and science. There are lots of problems attached to this. Goals for integration are put forward very tentatively but with some examples of ways in which integration could take place. In the long run I don't think that there's any doubt that some form of integration will take place in lower primary school, particularly when people begin to accept the idea of a non-directive lower primary education, where children learn by playing and doing and by handling educational materials.

Q: *What are your major plans for the future?*

A: In the Cambridge Conference itself, nothing. But we are presently trying to develop a proposal to initiate experimental activities within the state teacher-training colleges.

Project Title: Stanford Computer-Assisted Instruction Projects
Location: Stanford University, Palo Alto, California
Director: Patrick Suppes

Background

The Stanford Center has been working with various computer-assisted instruction courses since 1963. One of the interesting aspects of the elementary school mathematics developmental work is the notion that a tutorial CAI program can be developed from a drill and practice CAI program. Such an approach allows for reasonably rapid CAI course development, collection of extensive data on computational difficulties, and exposure of large numbers of children to CAI. It has the fault of concentrating on a facet of the curriculum that, with effort, teachers can teach effectively, while it fails to provide help for children who are having difficulty at the conceptual level.

The project has changed in scope and direction since its inception. At present it is the belief of the project staff that the use of reasonably inexpensive teletype machines can accomplish the drill and practice task. Previously, more sophisticated and expensive equipment was employed in the project.

The project has been widely reported in professional journals, popular magazines, and the press.

INTERVIEW with PATRICK SUPPES, Director
STANFORD COMPUTER-ASSISTED INSTRUCTION PROJECTS

Marilyn N. Suydam and C. Alan Riedesel
(3 February 1969)

Q: *Shall we start by discussing current projects that you're working on?*

A: Sure. We have a progress report with a description of all the projects here. The one that's now available is the one that ended on September 30; we're just preparing the one for the next quarter. So I'll give you a survey of all the projects here.

In terms of what you saw this morning, the drill and practice supplementary mathematics program, the main thing that's happening is that we are now changing the structure of this. I think you heard Max Jerman talk a bit about it. We're amassing reorganization material into fifteen strands, running from counting and place value, horizontal addition, vertical addition, horizontal subtraction, vertical subtraction, on through equations, decimals, C-A-D. laws--that is, the commutative, associative and distributive laws--problem solving, and word problems. Each strand is organized into classes on the strands, like beads on the strand; each bead or class, which is a collection of problems of about equal difficulty, has a grade placement. The student then proceeds on each strand individually and separately from all other students to make his way through this structure of fifteen strands, with each one calibrated to either a tenth or a half of a tenth of a grade level. You hear a lot of examples of that. For instance--if we just put the chart out here--there will be counting and place value. Here's what comes under the fifth grade-- 'Is *a* a prime number?' at 5.0, problems of that sort. 5.10-- 'Is x a factor of y?' In other words, the elementary number theory comes under this counting and place value. 'Is x a multiple of y?' 'Is a a prime number where now the range of numbers is larger?' And so we've organized this way, because it's a fairly massive effort. We took three textbook series and looked at their grade placements of topics and then we've used also our own research. (You may be familiar with these linear regression models that we've been analyzing a lot of data on.) So we took a mix of the textbooks which give a normative idea of placement, our own research on difficulty, and then some intuition to smooth things out so there aren't discontinuities, and checks that nothing's been left out, and we are just now completing the organization of the entire core curriculum of grades one to six. We looked at our data from the past three years on all problem types, and we have for the average student, an estimated probability correct when he works one of these problems. Using that probability correct and looking at the assignment of the percentage of the curriculum at each grade level devoted to each strand, we then computed, granted the time the student will be on the teletype, criteria for each class so that the ordinary student should make one year's progress in one year. That's a probabilistic calculation in terms of expectations. That kind of detailed calculation is not a traditional part of curriculum work. We don't expect all those calculations to be correct the first time, but the point is that we have an analytical and statistical inference base to go right back in and correct the structure. So we're really enthusiastic about this. We feel we've got an analytical grip on something that far exceeds our ordinary intuitions about curriculum, I mean in relation to what we've done in the past. Our present target is to have that operational early in March.

We are now doing two additional things, well really three or four additional things, all built around this. First, we're extending into grades seven and eight. So we're looking for where these strands extend into seven and eight. Now it turns out (and we plan to run on into algebra) that about two-thirds of mathematics textbooks for grades seven and eight can be carried on in the natural way. The things we don't carry forward within this environment are sets, geometry, and some of the graphical work. Excluding that material, about two-thirds of the curriculum books--in other words the total curriculum--is included. Now the step beyond that, we'll go through grade eight--a completely homogeneous one through eight and each child can get into it and be according to his own skill level. So in the same class you could have students spread out by three and four years. On top of that, we're placing algebra; we're just now in the process of analyzing ninth grade algebra. We're going to build that on top, then tenth grade algebra, and at that point we'll be relating it to our logic and algebra program which is a separate tutorial program I haven't spoken about. So our objective really is to get a large part of the K-12 curriculum, with the exception of geometry.

We are also now beginning some experimental work in the geometry because for that purpose we must have illustrations. We are going to start that in the elementary school and we're writing what we call an experimental strand that starts with grade one. We'll carry geometry through using an off-line collection of figures and even physical objects--a box of stuff that goes with the terminal for the students doing geometry. Now that's just in the process of preparation, but we hope to implement that on the experimental level. When that is implemented, then we'll move that on up through the grades too. So in other words, we're really trying to organize a quite analytical structure over a large part of the school curriculum.

Q: *And this will be, even as you go on, a drill and practice motif throughout?*

A: We're organizing a drill and practice, but then our long-range objective is to come back and make it tutorial. Now we'll probably start with tutorial. We already have a logic and algebra tutorial. That's a separate program that starts in either grades four or five and is aimed at bright students. The other program is aimed at all students. The algebra is already tutorial and goes through elements of inference and then into a fairly good axiomatic treatment of ordered fields, so it's a tough body of material. In fact one of the things we're doing is gradually making it simpler, so it is not too tough. The students in that get a very thorough introduction to proofs--I mean the accent is on proofs and learning how to give coherent, correct mathematical arguments. That curriculum we ultimately may take into the calculus--going out of algebra into the calculus on an experimental basis--but again that's a plan that hasn't been realized. At the present time, that is a three-year curriculum--the logic and algebra project.

Now we turn to tutorial effort: the first thing beyond the logic and algebra that we'll try to make tutorial will be the geometry. The reason for this is that ordinary elementary school teachers--and in fact too many junior high math teachers--do not have a very extensive background in geometry--that's particularly true of the elementary school teachers. They feel very shaky teaching very much geometry and we feel that that's the place we can give them assistance by beefing up what we offer in curriculum materials.

I've said it here rather quickly--it's a fairly substantial effort that will keep us occupied for several years. But it does give you a sense of where we're going. Now you'll be interested in the teachers' reports that we're going to put out under this strand program. What we'll print out is--on a weekly or bi-weekly basis--the grade placement of each student in the class for each strand. We think that will be a very meaningful piece of information for the teacher. And then we'll also give her the averages across strands for her class and tell her which strands the class averages the lowest and the best on.

Q: *Will the teacher try to keep the youngsters balanced among strands?*

A: The program controls the exposure.

Q: *In other words, if I move like mad in numeration but I'm poor on multiplication, then I'll be hit heavy with multiplication.*

A: Hopefully, the teacher will give students additional work where the program shows they're weak. So the information for her is for diagnostic purposes.

Q: *But the order in which the students get the program is computer-controlled?*

A: That's right. In fact the order of the program is random. What we've decided is that it's mixed drills continually. We've decided that to keep them in a 'ready' state on everything is a desirable way to do it.

Q: *So that when you go on you aren't really sure whether you're going to get strand one or strand two?*

A: That's right. You get one problem, then you get another problem. We feel from looking at some old data and some data of ours that this business of being 'at the ready' in several directions will be a desirable feature. Let me put it this way: too much data is carried in terms of prediction by what we call a sequential variable, independent of the type of problem. If the problem was like the type of the one preceding, the student's performance is very good. In other words we're not in that environment testing the students as thoroughly and exposing them to sequential material.

Q: *But aren't you going to get some confounding here of what's going on in their regular classroom?*

A: Oh, sure. See the idea is, Marilyn, that for the average student the program will be back of the class a little, so the average student will be running back of where the teacher is and it's like a continual review in the way it's conceived. Now, the only real problem will arise with the bright students who get ahead of the class. There can be some problems there and frankly that's one of our points of investigation. The slow students, though, will have had all of the material already presented.

Q: *But you're still going to have some problems in the elementary school; in particular, where one teacher is teaching the unit on geometry and the next teacher is not teaching that material at the time when two students get it on line. The student having it in the classroom is going to be getting reinforcement from the classroom experience.*

A: Well I think, the way we conceive of the drill and practice program, the maintenance of skills is almost as important as learning them, and it's really aimed at the maintenance of skills. It's really aimed at picking them up and it's only accidental, particularly with randomization. It will be only a small percentage of problems on any day that will be exactly what the teacher's talking about. And the aim is to keep all those skills at a pretty good state of preparation. Because you know how it is, particularly with youngsters in third or second grade, you can teach them one week and the next week they've forgotten. And the point of the program is to provide the teacher support in terms of maintaining skills.

Q: *I think this makes better use of drill and practice, than the typical 'all multiplication,' 'all . . . .'*

A: Yes, me too. You know there's a lot of evidence on that. It goes back to the twenties. Guy Wilson, for instance, did a lot of studies.

What's back of the decision not to put the option in the teacher's hands is: there are fifteen strands--if she has 30 students, that's 450 decisions daily. That is too much decision-making for the teacher to take on. What we felt is what really the teachers want is a program that's very well articulated, that'll maintain these skills, and they don't have to worry a bit about it. It takes care of that aspect. We want to take that continual, daily decision-making.

Q: *But if she does want to override the computer--can she?*

A: No, she cannot. In the present set-up she cannot.

Q: *I agree. It's too much to expect a teacher to make that number of decisions a day.*

A: And you see, in order to have her override, we'd have to set up a full procedure as to how she'd override and we haven't done that yet.

Q: *I had another question earlier. You used three textbooks--why three? What three?*

A: We picked three because it was a manageable number. We went through three complete textbook series and we picked three that we thought were somewhat different. Now, as you might expect, egocentrically we picked my own *Sets and Numbers*. We picked Addison-Wesley, and then we picked a more conservative series, Harcourt, Brace and World. So that gave us a kind of spread, and our aim was to get some spread. I hold no brief in any deep way. In fact, what we find actually is, in gross terms (it's probably true of elementary mathematics almost more than any subject), a remarkably consistent agreement. You can find differences--I won't say they aren't there--they are there, but in terms of the main-line stuff like addition, subtraction, multiplication--great agreement on general grade placement.

Q: *In geometry I imagine you found differences.*

A: Geometry's another matter. Geometry's all over the map. Some of the older series will have practically nothing on geometry, so geometry's something else again.

Q: *The material developed for the first and second grade: how does that fit in with the tutorial material?*

A: Ah, you mean the Brentwood stuff that we developed, the old Brentwood material. Did you ever visit old Brentwood? Ah, yes. Tell me what was your impression before I answer that, of the atmosphere of the two?

Q: *Of the atmosphere when the children are on-line?*

A: Yes, on-line now and when you were at Brentwood before.

Q: *Well, I think there's no real comparison because before those children had been on-line for a period of time and many of the things today were the result of newness. They weren't used to the system yet. Interest was by and large good. Last time around a few who came in really weren't interested.*

A: The only reason I ask you is that our impression is that the drill and practice material holds the interest a little tighter than the Brentwood system did. You know they're on a shorter time. I was just curious if you got any feeling of that comparison.

Q: *None that I recall really. They're just two different types of systems and they operate so differently, besides the content being so different.*

A: Of course as you know Brentwood's only half operational now, the other half's going to be reading which is coming up so it'll be a busy place when it's fully full blast.

In answer to your question--we are using that first grade material in a variety of ways without lifting it wholesale. I mean in other words, the systems have changed enough that there's nothing goes over intact but we're particularly using some of the geometrical material.

Q: *That's good because when Karl (Anselm) said the developmental program was 'in limbo,' I felt it was too bad that developmental time going to waste.*

A: But I will say this about developmental work and technology--one thing you've got to be willing to do is to cut your losses and change. You know what I mean?

Q: *Oh yes.*

A: More so than in most work.

Q: *Except the cost is so high that you hate to see this.*

A: I agree. But we're using as much of that as we can, using that experience.

Q: *We've had some experience with a spelling program that we developed under Title III that worked very, very well at the sixth grade level only it was on the old 1050 typewriter system with a tape recorder and image projector. Of course when that went out, the program just stopped.*

A: I'm still a very great advocate myself of typewriter or teletype type terminals because of cost.

Q: *Well, it makes a real difference.*

A: You know in terms of schools really being able to do something . . . .

Q: *Yes, in terms of just plain experimental work you get a lot of data that way.*

A: You get a lot of data and you know now we've got enough data--we're awash in data. It's good though. We get a lot of validity of findings from it that are very satisfactory you know.

Q: *What's the philosophical basis of most of the CAI teaching as it's related to the type of teaching a classroom teacher might do?*

A: Ah, what kind of orientation--I'm not quite sure . . . .

Q: *Would you say that the approach taken is more of a didactic approach, more giving explanations, more of an approach where the youngster's trying to make the next jump himself, or somewhere in between?*

A: Well, I think we have to distinguish types of programs.

Q: *Yes--this might not be an appropriate question for your project. But, for example, one group will say 'we have an eclectic approach' or someone else will say 'ours is heavily discovery-oriented.' I* *don't think the terminology is quite as appropriate for you but I'd like to discuss it.*

A: Sure. One of the things for example is in the logic and algebra program which is essentially a dialogue, because we accept within a language any input they give. That is, they could put in any command. So anything that's within the language is accepted as an input of theirs. Then we encourage the students to go in any direction they feel has a chance of making it on a proof. A very fundamental thing about that program is, it doesn't check down alternative courses or proofs. There's no listing or storage anywhere of proofs. What it does is make a recursive check simply on the validity of each step. If the student comes out with the correct proof, then they'll accept it regardless of the way it was reached. You could have a thousand different proofs and it would be no problem for the system. So it's unusual in that respect. It permits a diversity of student approach that is not possible in the drill and practice work. Now we have some of that already in the drill and practice program and our problem-solving because we took a leaf from the logic book. In the problem-solving the students ask the computer to do all the computational work, solving word problems and they can go at that any way they want to. When they get a number they think is the right answer, they hit a key and show that's the answer and the computer will say yes or no.

Q: *So your problem-solving isn't really aimed at writing a correct mathematical sentence but at solving problems.*

A: That's right.

Q: *In other words, if they do it as an addition fine, if they do it as subtraction fine, ...*

A: Right, right.

Q: *But that means then that you're collecting no data on how they're answering?*

A: Oh, yes.

Q: *On how they're proceeding?*

A: Oh yes. We have a record of that. In fact I've got a paper on my desk there--the first paper on that. I have a girl writing a dissertation on the analysis of the types of errors they make.

Q: *What ways do you consider individual differences throughout the program? You mentioned, certainly, some of them before.*

A: Operationally, of course it's clear how we're considering individual differences, that is, in the strand structure we're moving toward and in the block structure you've been watching. Of course there are various levels of difficulty. The strand structure is another step beyond that. That is, each student is placed in a strand structure with grade placement on each strand, totally independent of any other student. So there's individualization at the one-one level--it doesn't matter what his fellow students are doing.

Q: *How are you determining where he is to be placed?*

A: The only problem is initial start. Once we put him in then we have criteria for passing through each class and he has to pass criteria.

Q: *What do you set as criteria?*

A: Criteria have been computed from this looking at the expected motion of the average student.

Q: *But suppose that criterion--the initial one--happens to be incorrect?*

A: For placing him? It either goes up or down, by the way. You know he can also go down. So a student having trouble is pushed down. The thing about the system is it's made so every student can find a place he can work. If he has to sink to the beginning of the first grade, he can.

Q: *But suppose the initial criterion says he's placed at 1.5 and he was sick that day--he actually belongs at 2.5. Can it make that adjustment?*

A: Then he'll go up very quickly.

Q: *But he has to go through the material. He doesn't go around it.*

A: Under the present set-up he must go through. But the student who has a mastery of all the material can go up quite rapidly. It is set up right now exactly that way.

Q: *You made the decision to do that or is it a limitation of your computer?*

A: Oh no, we made the decision to do that. You see I think the problem is, Marilyn, that for any actual decision there are always several alternative routes. It's a very good question, you see, as to what are the precise arguments back of the actual decisions taken. Certainly one thing that has motivated us at first, because we don't understand the mathematical theory of the student's behavior better than we do, is to keep it relatively uniform and simple to see how the data look and how the progress of the student shows up. On the basis of that we can then begin to introduce wrinkles.

Q: *I asked that particular question because I'm working with equipment where they're telling me, no you can't do that.*

A: Oh well, it will be easy to do that. If we want to put in diagnostic tests that, so to speak, re-place the students, we could do that and do it wherever we wanted to. It wouldn't be a problem. And really it would be our decision whether to insert those diagnostic tests and to jump them. Our own feeling from a lot of experience working with elementary math is, if a kid's too high, he's going to fall quite rapidly. If a kid is too low it's not really a problem because he can move fast and students do not at that level mind working problems. In fact they get a certain pleasure out of a lot of easy work. You know they'll go right through it.

Q: *If they can go fast enough--yes.*

A: Yes. And it's set up so that the maximum speed can be pretty good.

(Interview with P. Suppes -- M. N. Suydam and C. A. Riedesel)

Q: *What do you find in terms of in-service work with the teachers? What's their reaction?*

A: Max Jerman and I just have an article coming out in the *Arithmetic Teacher* on our work in Mississippi and I think in many ways that was one of our more successful in-service jobs. I think the problem is very complex. We all recognize this. I mean particularly in the upper three grades of the elementary school--grades four, five, and six--the ordinary elementary school teacher has very slight orientation toward math--the ordinary teacher, not the ones who come to N.C.T.M. meetings. She would just as soon have as little to do with the math curriculum as possible. One of the problems we find is we have to be a little careful with some of the teachers --they'll let the drill and practice program take over the math program. We have to emphasize that this is not meant to take over your instructional program. This is only five minutes a day or six minutes a day. You've got your own program to worry about for another 40 minutes. Well, I think we all know that if we went in and did any timing with most of these teachers who are not very happy with math, they're slipping and sliding that amount of time in the math curriculum continually. They themselves don't feel too comfortable with the material. They don't like math. One of the things we've tried to do is to get these teachers involved with math in a general way. I would say we have felt that's as important as anything--to give them a kind of Renaissance feeling about the role of mathematics in their teaching. Sometimes we're successful and sometimes we're probably not. I mean that's again the real world of elementary school teachers. But, I think it's fair to say, we have had a very clear experience that it is very desirable to at least have some sort of workshop to explain to teachers the structure of the program, the purposes of it, what it's supposed to do, and what it's not supposed to do, so they feel at home with it and understand the role of the program in their teaching in their school.

Q: *You indicated that some teachers tend to sometimes slack off and let the computer program take over. Would there be more teachers like this or more teachers who would feel, 'Well, I'd better take care of this myself just in case that computer isn't doing it'?*

A: Well, I'm not quite sure how to answer that. I would say the main thing we have noticed is that teachers who get involved in it do use in an effective way the reports about the students, what the students are low in.

Q: *Is there a halo effect here?*

A: Well, halo effect in a sense. More importantly they really try to use the information. One of the things we've done in teachers' reports under the present set-up is when individual students take a bad fall from the previous day or two, we print out their names to the teacher. 'So and so's performance dropped by a significant amount' and she has a list of those students. The good teachers then will nose into what's going on with that kid--is it a home problem, is it lack of interest in math, what's happened to him those last couple of days?

Q: *The good teachers.*

A: Yes.

Q: *What percent?*

A: Oh, I couldn't give you a serious estimate. I mean that's a very complicated piece of research to put a number back of and to say also what percentage of the time. I think the honest thing to say, Marilyn, is that we don't have any hard data on the way in which the teachers use our reports. I think it's a very useful thing to have hard data on. We have the anecdotal data, we have the fact that they want the reports, but what they actually operationally do with information about the students, we do not have. It really needs to be made a special object of research and a rather important one too.

One thing we have gotten out--did you see our book that came out in 1965-66 on computer-assisted instruction where we have teacher questionnaires, parent questionnaires, and student questionnaires? At least it's questionnaire information--there are three chapters. That was published by Academic Press last August, 'Computer-Assisted Instruction: Stanford's 1965-66 Arithmetic Program.' That does have a lot of data in it.

Q: *One of the impressions I carried away from here last time was that there really wasn't too high a level of correlation between what was going on in the classroom in their arithmetic lesson with what you were doing on the CAI program--and this was the tutorial program, wasn't it?*

A: If it was Brentwood it must have been because we totally controlled the whole thing--the ordinary teachers did not do anything. They had no responsibility.

Q: *Well, this was just their general attitude toward what was going on out in that building.*

A: Now you see that's special to the tutorial. That is not the drill and practice environment where we emphasize we are not replacing the teacher's work. On the other hand we are following back to the average student to the teacher's work.

Q: *I gathered at that point it was a function of having a separate building there (at Brentwood).*

A: Yes. That makes that a somewhat special environment. That's the only school we operate that way in.

Q: *You are finding that putting the terminals right within the regular building is best?*

A: In every school but that they're in some kind of room in the building.

Q: *In the classroom or . . . .*

A: No, not in the classroom. By the way, we might speak of that if you're interested. We have two modes of operation that we've tried extensively. One is to put one terminal in each classroom and the other is to have the classrooms grouped in one place in the building and then to put students through in groups. We find the latter much more efficient and really probably more acceptable from the teacher's standpoint. The teacher has to worry too much about maintenance and use of the terminal when it's in her classroom. I think really my philosophy is, to make it explicit, what we should do is take a load off the teachers by providing a quality control, review and maintenance of arithmetic skills. The average elementary school teacher does not want to do that. She does not want to do it in a way where she really looks at the data of what the students are doing. She wants somebody else to do it. It's a very good thing for a computer to provide. The exceptional teacher will want to be involved up to the hilt but I am very much convinced that is the exceptional teacher, from a lot of hard-tack experience in ten years. My philosophy, Marilyn, is really to think of it as a deep teacher aid, something that's going to take a load from the teachers. We'd like to make it in that sense: it'll run, if teachers give a damn or don't give a damn, it's at least going to go on an organized basis.

Q: *It's a bonus if the teacher does relate it to her classroom work.*

A: It's a bonus--it's certainly a bonus. We put out a lot of detailed information about student behavior in math. The good teacher could use that very effectively to pinpoint work but we have no way of knowing what they do.

Q: *You made the decision that you'd provide drill and practice on one particular thing. Isn't there some reason for including a little bit of teaching in there though, when the child's answering a question?*

A: Well, that changes the name of the game entirely. That's exactly what our tutorial programs do. For example, when a student makes a mistake in the logic program, he's not only told he makes a mistake, he's given an exact diagnosis of the mistake. But that changes the character of the program. The drill and practice program is set up to handle and process quickly lots of students. It's a matter of the complexity of the program. It does not attempt to diagnose the character and nature of the answer given by the student when it's wrong. Again, that's a practical decision based on hard-tack experience. It's not that it isn't desirable to do that; it's a very complicated world in which you do that.

Q: *Either way you have some confounding of data though don't you?*

A: Oh, whatever you do you have confounding of data.

Q: *When the youngster answers incorrectly because he doesn't understand the instructions, that gives you a different kind of data.*

A: That's true of all test data or behavioral data.

Q: *What I'm basically asking, is this a decision that was made because of data collection or was it a decision made that it's best for the child, that he will learn better what you're trying to teach him this way?*

A: I don't think that we have a hard data base to say it's best for the child. I think the question's open as to what's best for the child. I don't think that question's at all clear; that is, if you have a fixed number of minutes to tackle a problem, what's the best way to use those minutes is a very complicated, sophisticated inference. Whether it's better to go ahead and tell him he's wrong and give him another problem or whether to linger and analyze that problem in a kind of supplementary program. So I don't know the answer to that. The decision here was based upon its character as a supplementary program, the depth of processing we wanted to devote to each student on-line. That's what's back of the student--not any claims about optimization. But I want to state very clearly:

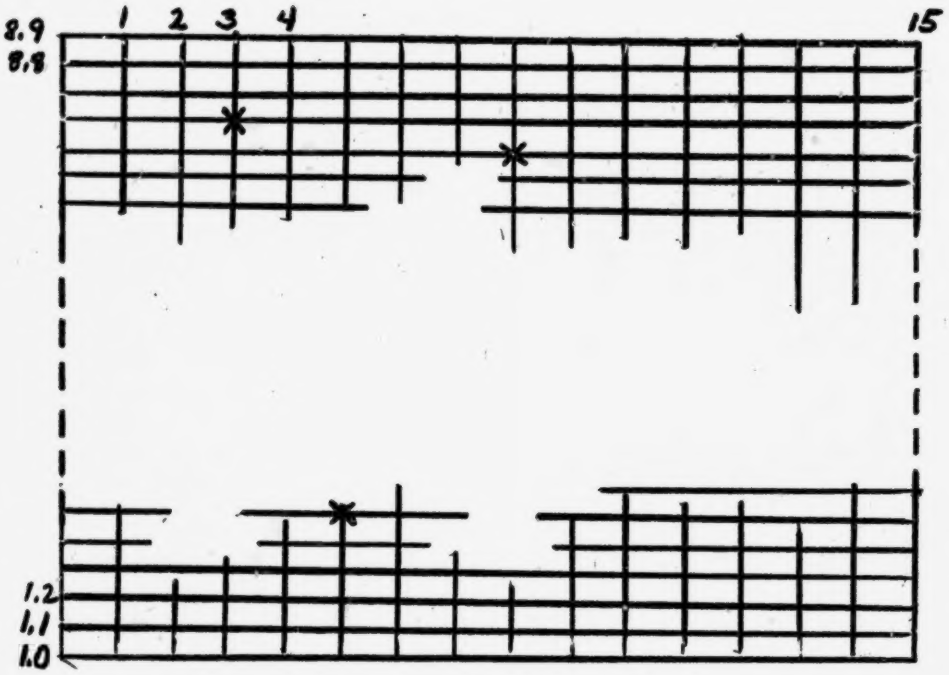
I don't think any of these questions have research answers as yet so one has to sort of 'bull ahead' operationally with curriculum. Now, you see, the whole point of our tutorial efforts is to move just the way you're talking about. The problem is partly one of expense too. It's very much more expensive, so it's not just a matter of what's best, it's a matter of economic trade-off. In our logic curriculum where we come back and do that it's just a lot more expensive. In our environment that's not critical, but for thinking about people using it more generally, it is. There we come back to try to diagnose what the problem is, tell the student how to apply this rule but there's no proper antecedent. That's what we're going to do in our general tutorial as we pick up various strands. We did that at Brentwood: we tried to do some diagnostics there of responses. But it is an expensive proposition. In other words, generally speaking, other things being equal, I'm very much in favor of it.

Q: *Other things being equal, were funding not a problem but you had to go one way--either drill and practice or pure tutorial--which way would you go?*

A: Well, you see, Alan, I'd want to give a sort of mixed answer. One of the things that's characteristic of curriculum work is that it is very hard to get an analytical, global analysis of the curriculum, where you're looking at parameters that are estimated in a standard statistical sense that cover any amount of time. I mean, whether it's the small individual problems or segments. One of the virtues of a drill and practice program is we can get a landscape approach that is rigorous and statistically meaningful, and we can build models theoretically for it in a way that's very difficult on a tutorial program. So I think from a research standpoint we have not yet plunged the depths at all on the drill and practice. So I have an interest in doing that, in the sense that we have a whole set of models and estimation of parameters. Furthermore, it requires a delicate adjustment of parameters to have a notion of the student route, and I think it's hopeless to get right now in tutorial. We do tutorial by the seat of our pants. We can do drill and practice with some scientific understanding. Now for the long run, I'm very much more in favor of the tutorial. Even though we're just beginning to run this, we're almost over the hump on the drill and practice. That is, this is the last big effort I intend to make on drill and practice. We're already starting to dig in now on tutorial. It's appropriate that we do that here where we are concerned in a direct way with cost. It's a very complicated thing--it's going to take us a long time to learn how to do tutorial with a computer but we think the possibilities are great. The technical problems we have now solved which we hadn't solved before. You see the problem with the Brentwood material was that every message you wanted to say had to be prerecorded. Now you see we now have a new technical set-up; with the channels and base vocabulary, we can be very contingent on the students. So we've got the power now from a technological standpoint to really get serious on the tutorial and we've already done that--we've already got some very interesting ideas in the logic program, that is, we have written a general dialogue program. You can push any problem into it and the students work on it. It's written in such a way that the program can look at his work and ask him detailed questions about what he's doing, to give him leading hints. We understand how to do it in the logic, we have a beginning of understanding how to do it in the algebra. We haven't really tackled hard yet the other areas, but that's the direction we're going to be moving. Those are the problems, of course, from a broad standpoint which are really challenging and very complicated.

Q: *Do you think that before really seriously getting scientific in tutorial, you need the learning theory models such as you developed from analysis of the drill and practice data?*

A: I think the thing about the drill and practice data on this strand thing is that it gives us a framework in which to think about tutorial. We know the kids should be doing this and this and we have some landscape view now of what we think the skills ought to be and the calibration of time on it. So for example, let's take an instance. We can look at these fifteen strands. We can say, well, now, obviously we can't create a tutorial across eight grades instantaneously for these fifteen strands. You know it's a mammouth, complicated enterprise. Let's ask ourselves first--where are the teachers least prepared? Well, what we've come out with on the general strands, teachers are least prepared in geometry. The first step in tutorial will be to see if we can walk right up through the geometry strand. Then the curriculum question is, where is the next area? Well, we have an alternative strategy. As we look at the drill and practice, where is it that we see the students getting hung up? Let's go in and try to put some blobs of tutorial in what are obviously tough spots--where there is a sharp transition change in concept and difficulty. So, another proposal we're examining is to do that--look for spots that have mixed structure. We have fifteen strands--every strand doesn't go through every grade level, but there are roughly at least ten classes per grade level so it's roughly a matrix from 1 to 15 and then all the grade levels; here are 1, 1.0, 1.1 up to 8.9.



So here's a spot that's troublesome, here's a spot and here's a spot.

Q: *So instead of going across the strands, you'll just pick up those spots.*

A: Exactly; that's the real change. Instead of doing it by grade level we're doing it by conceptual strand. We're doing it vertically rather than horizontally.

Q: *At this point in the seventh grade classroom, aha, we plug them into the CAI program for their individual student needs at a particular point. That should be a real plus particularly with the seventh grader at the fifth grade level on one topic, where the teacher wouldn't bother to go over and give it to him anyway.*

A: Well now you know I'll tell you something. The honest to God's truth is you know I don't quite know what's going to happen. I mean it's going to take some experience to see. There's no doubt from the standpoint of the program and the organization--at the drill and practice level now but as we tutorialize it too--the students can fly right ahead of the class--the very bright students. What's going to happen when they do--I mean I just don't know what the actual thread will turn out to be with a class running for a couple of years.

Q: *It would be very interesting, too, if we could possibly see what happens in a classroom and how the teacher reacts to this sudden burst.*

A: Let me say one thing that is very interesting on this. We already have one radical kind of experience because of this logic program which we've run with bright kids in the elementary school all around the country. If we run it where we don't have any direct contact with the students, it comes in on the terminal like in Mississippi. You get quite diverse reactions from the teachers. Some teachers we have, boy, they're right there every day, at least two lessons ahead of the students. They have to get there at 7:30 to get on the terminal so they're ahead of the students. The teacher will be ahead of the best student in her class or his class. Other teachers couldn't care less. They just push it completely aside and the students have to fly on their own. I think that w would be the same thing here. Some teachers will be very much on top of what's going on; for other teachers, the less they can be involved in it, the happier they'll be.

Q: *Then there's even a spectrum of teachers who will be derogatory of it because it's upsetting their applecart because a youngster is ahead of where they wanted to be at the moment.*

A: I don't think that's out of the question.

Q: *I don't know what percent it is; I hope it isn't a very large percent but I would guess that it will happen.*

A: Well, I think that what we're going to advocate is for quite a deep level of individualization--deeper than in the present structures. There's no doubt that if it's successful it makes certain waves in the classroom set-up. I can speak of this better about a year from now.

Q: *You won't know whether it's really individualizing until you do go across a strand because in this case many teachers are going to just plug the whole class in at one particular point.*

A: Oh, it doesn't matter if they put them in at the same point. I don't care about that at all--it's like all horses in a race starting at the same place, Marilyn. In two weeks they'll be at completely different positions.

Q: *But then they have a point where they stop though because you don't have material written beyond that.*

A: Oh no, oh no. We'll be ahead of every student. That's a commitment of ours. No student stops because there's not material.

Q: *So you may begin writing at this point but then you carry it right on through and up from there.*

A: Right.

Q: *Oh, okay.*

A: In other words, every class will settle into its own distribution spread. The thing we're terribly curious to see actually and we're sitting here really on pins and needles. We really have the opportunity in an unusual way to observe--because we have a large population of students, we have a powerful computer system now organized for this--to see really how these four or five thousand students are going to spread out.

Q: *Are you just going to run it here?*

A: No. We'll run it in the California schools and almost certainly in Mississippi too. We are just starting computer-assisted instruction programming--it's going into a high school, perhaps in San Francisco.

Q: *Karl was talking about that.*

A: Right. That's supposed to start up Wednesday and right now we're in search of the computer. We need a small computer for line processing in that other school and we're searching for one. The curriculum is in very good shape.

Q: *What do you think the major impact of your various projects has been?*

A: Yes. I view computer-assisted instruction as having, roughly speaking, three stages. First is the development around university centers where there's a lot of various types of technological and technical and research talent. The second stage is to have demonstration projects in school systems. The third stage is where school systems make this an integral part of some part of their curriculum. We're not really at stage three anywhere with the possible exception of Mississippi. They seem to be going for broke--they're going on their own now after their experience with us.

Q: *This practically demands individual funding.*

A: Yes, and they're pushing to get into it on that basis. They probably will. But with a few rare exceptions we're somewhere between stage one and stage two. I think what we've done is had influence on school systems getting into the area. My own feeling is that in many ways from the standpoint of curriculum what we're doing is, when one looks at innovation, really something rather conservative. I mean conceptually we're trying to bring an individualized instructional package that deals with curriculum that everybody accepts as part of the kids' education. Our problem is to show how it can be done effectively. I think there's a second stage that's going to have to be faced up to, how can it be done economically. It's still a problem certainly.

Q: *Do you see any dangers from improper use of materials or misinterpretations?*

A: I don't see any dangers peculiar to this field. I think those dangers always exist in curriculum work. I don't think anything special I guess is what I would say.

Q: *Do you run into this type of thing: we're running the drill and practice material and Suppes is known to be very modern in modern math so this must be modern math so this is what I'm going to do every day in the classroom.*

A: Actually you see the drill and practice program reflects an instance of the core body of material. There's practically nothing in the drill and practice material that's not in every standard elementary school series really. Now the logic and algebra is something else. So there's a different response depending on the program, Alan. The logic and algebra program is a much more radical program in terms of the ordinary curriculum content of elementary school mathematics--so somebody might wish to say we're corrupting the students with logic.

Q: *In the literature, you run into large numbers of articles now about the integration between science and math and also a push back to applications--applied mathematics as opposed to pure mathematics. What's your feeling on your projects concerning the amount of application--of where application fits in and where science fits in?*

A: Well, of course, I don't think one can get a very heavy dose of science in the elementary school math. Science just is not set up that way yet. But I'm very keen about the applications. I think, frankly, the most exciting part of the drill and practice program is the problem-solving part where the students are given the computer as a tool. Their problem is just to read those problems and understand them. We all know that's a classical problem. We're going to have the same approach to the algebra. In other words, we feel that that's really the best part--the most exciting part of the drill and practice program--the emphasis on applications. We also have put a tremendous body of material--partly because of the known deficiencies of the American students--on problems of measurement, so-called problems of denominate numbers. We've put a considerable interest in that program on applications just because all the test data show

how American students do not have good command of that material. But intellectually it's the problem-solving thing that I like. So I'm much more applications-oriented myself.

Q: *This seems to me to be, because I've always been very applications-oriented, an interesting thing--each of the persons we've talked to --I guess with the exception maybe of David Page--felt a real swing themselves toward application. I would have thought Dienes wouldn't be interested in applications at all, and he seemed to be quite interested.*

A: As a matter of fact, if we weren't so involved in other things, I would really enjoy working on the relations between mathematics and elementary science programs. I think mathematics is so much more serious than the science program that the mathematics can be used to give the science program body. Something really very beautiful could be done in terms of integrating the two.

Q: *I was disappointed in terms of A.A.A.S. because the math is separate. It isn't as applied as we probably teach it in most standard elementary math programs; they feel the need to put it in there and yet they haven't. I suppose it's just a matter of timing, but to get a synthesis seems necessary.*

We were talking with Karl on the matter of evaluation and the fact that you're using the Stanford Achievement Test for the drill and practice program rather than an experimenter-made test.

A: Yes. Well, the reason is pressure from school systems. I mean I don't like the Stanford Achievement Test. You know we think a lot of the items are terrible.

Q: *Your own would be so much better I'm sure because it hits on the content that you're teaching.*

A: And the reason we use it is because it is normed and it is a standard test. Frankly, we think it's about as good as any of the ones on the market but we don't like it.

(Interview with P. Suppes -- M. N. Suydam and C. A. Riedesel)

Q: *I guess this doesn't have anything to do with this, but this has disturbed me very much. I've taught a course on research in elementary school mathematics where we're analyzing various research articles. At the beginning of the course almost every student (doctoral students with a reasonable amount of measurement and statistics) will give a greater quality of goodness toward the measurement instruments used if they're standardized as opposed to experimenter-made, even if it's a study of division of rational numbers. In fact, if you get significant differences on a standardized test on a topic such as that, it's due to something else other than the treatment anyway because there are only probably three items on that. It seems to me that this is something that those who deal with research have to communicate with the lay audience.*

A: I quite agree. We've been discussing this. I'm just now writing up, in fact, the evaluation of three of our programs--the drill and practice program, the Brentwood, and the Russian at the University level. We have used very extensively the Stanford Achievement Test in elementary schools. We find some of the items sort of hair-raising in relation to what we're teaching. We're not satisfied with it at all.

Q: *You can't use both?*

A: We could use both and part of it is just deciding to mount the effort to do that and to standardize it in our own framework. Once we stabilize this drill and practice, we hope to do some standardized testing. In fact we've been looking at the Stanford Achievement Test very closely. We even think we might write a report on the standard achievement test--not just the Stanford but several of them--on the basis of a detailed examination of items which I think would be useful. We're in transition on this, Marilyn, is a fair thing to say.

Q: *Another thing in terms of the evaluation: are you collecting data and/or are you going to look at data in terms of what programs these kids have in their school, and the difficulty levels which are showing up from the drill and practice materials?*

A: In the work we're just reporting now, no, because we're giving a very classical general control group with the Stanford Achievement Test. We are not looking at a more detailed level of difficulties. Now I think as we stabilize this changeover, we'll be planning to do that.

Q: *Depending upon the textbook series you use, won't you find wide variation on the difficulty on things like types of subtraction depending upon whether the teacher was emphasizing inverses? And then if you take out that factor and analyze it without what they were taught, you might find out which might be better to teach.*

A: Those inferences are rugged though, to pin down. I mean I agree but . . . .

Q: *Some of the difficulties you quoted in that paper in Georgia could very well be a factor of the textbook series used by the kids that produces the difficulty levels which you found. I mean, a couple of the difficulties didn't look like they made sense. But, depending upon the previous experience of the youngster, it might be that your data are right and also that other inferences are right. But in other schools this doesn't happen because of a different curricular structure.*

A: Well, I think as one gets a more refined analysis, it's desirable to attempt to see what the sources of the differences are. But it's true, it's just that we can't do all the things at once. I'm not at all opposed to that--it's just we've got our hands full with what we've got.

Q: *Are you going to do anything in terms of geometry with trying different levels at different times?*

A: Yes, exactly. In geometry one of the things we're planning to do is design it to get some inferences about the appropriateness of grade placement in a more global sense.

Q: *Some things we may be able to teach the younger children better than older children.*

A: We have that definitely in mind, Alan. We want to randomize across the classes and where we place them because we don't have a clear idea of placement on that. In fact, one of our problems with designing is actually to set it up in a way that we have some reasonable inference procedure to that question. No, we're very conscious of that. And geometry is, I think, the ideal case. You know it's surprising how little information has been published on geometry learning in the elementary school.

Q: *Very little--a few theoretical articles.*

A: You know there's Piaget-type stuff but that's not really on curriculum. Now one of the things on the research side that you'd be interested in--(you're familiar with the linear regression work, for example my Georgia paper) is that we're moving beyond that to a probabilistic automata to represent the student performance. This is much more of a processing model. The student or the group of students depending on how we estimate parameters are looked at as a probabilistic automata who've been taught an algorithm, but an algorithm that isn't always correctly applied. We can see where the teaching of that process has failed by exactly where the parameter estimations come out in the probabilistic automata. Of course, if the automata were not probabilistic, they would do every problem correctly 100 percent of the time, so the whole point of probabilistic conception is to provide a method of finding errors.

Q: *Is there feedback on this just into the CAI program or is there feedback to the classroom teacher?*

A: That right now is data, research feedback.

Q: *But eventually do you see the use of it in terms of programming?*

A: Oh, certainly. That kind of information and that kind of conception of sources of problems for the students feeds right back into the classroom too without any question.

Q: *It adds to some of your optimal sequencing?*

A: Yes. And also you know even more on the steps of the algorithm, you get a direct estimate of parameters indicating where the students are failing--what it is that's causing the problem. And that of course is good stuff for teachers' editions.

Q: *Depending upon the mode of instruction available, whether the approach is deductively or inductively or however . . . .*

A: No, not necessarily. There are a lot of things that you're varying. For example, what are going to be sources of trouble, say, in learning the addition algorithm with column addition. I think that's going to be invariant, Marilyn, whether you approach column addition deductively or inductively. In fact you see I don't think anybody much does it in a really deductive fashion.

Q: *Instead of inductive-deductive maybe didactically or through asking questions is more appropriate.*

A: After the teaching has occurred either way, my conjecture would be that difficulties will in many cases be common. They have to do with the character of the algorithm rather than with the exact way that they're taught. Now if you want to be really to the nth degree hard-nosed, one needs to do a detailed study with n-manifold teaching methods for each algorithm. It'll be the 21st century before that kind of work is completed.

Q: *It seems to me that this is one of the facets of research that we could do much more with on CAI in terms of various instructional approaches on teaching the algorithm.*

A: Yes. You know the biggest source of variance in the classroom is the teacher and we can knock out that source of variance. All we'd have to do is to really look and say, here are four models for teaching this, now let's look and see what happens and where the errors fall.

Project Title: School Mathematics Study Group (SMSG)
Location: Stanford University, Palo Alto, California
Director: E. G. Begle

Background

Probably the most extensive mathematics project is the School Mathematics Study Group, which originated at Yale University in 1958. From its primary focus on the development of modern materials for twelfth grade down through kindergarten, it branched into large numbers of related activities. The following are representative of these current thrusts:

(1) *The development of programmed materials at the secondary level.* This is aimed at developing a more rational sequence, one in which algebra and geometry reinforce each other. Topics desirable for everyone are placed at the beginning of the sequence; technical ideas which only a few will need come later. Materials are accessible to all, though some students will proceed more quickly than others.

(2) *The production of films for elementary teachers.*

(3) *The development of primary materials for the disadvantaged.* The regular SMSG primary materials were modified to make greater use of concrete materials and firsthand experiences of children.

(4) *The development of junior high school materials for the disadvantaged.* These materials focus on relieving the computational burden of memorization from the low achiever by use of tables and, when necessary, calculators. Daily worksheets with summaries of the lesson and problems are used, rather than a textbook format. Teachers are guided by these worksheets to see what to do and how to do it. These materials have proven to work effectively, though gains in mathematical achievement in the second year were not as good as the first year; however, there were promising gains in pupils' realistic self-concept. (Ideals went down, while actual self-concept went up.)

(5) *National Longitudinal Study of Mathematics Achievement.* NLSMA is a five-year study (with an initial 110,000 students in grades 4, 7, and 10) which attempted to ascertain the effectiveness of SMSG materials. They compared pupils using SMSG with those using other modern textbooks and those using traditional textbooks. An important facet of the study was the development of extensive batteries of achievement and attitude tests. (These and the findings are available in NLSMA Reports #1-12.) Data are still being analyzed, and a vast amount of additional untapped data will be made available to other researchers. A major conclusion is that the effects are not as diverse as it was feared, but far more complex than most thought. Some modern programs are better on comprehension, application, and analysis; others are better on computation, as in general, are traditional texts.

They are fairly confident that replication would show the same results, since there is much internal consistency. The effect of individual differences on student achievement was analyzed in terms of particular classifying cognitive and affective variables, once the obvious I.Q. variable was factored out. It should be noted that probably a small percentage of the textbooks involved in NLSMA were affected by SMSG. New books are now more in the spirit of SMSG--it persuaded textbook publishers that there was a need and that they could sell books like those of SMSG.

(6) *The development of content background books for elementary and secondary teachers.* These have received wide use in institutes and in-service work. Also, they became the models for many of the current commercially published content books for teacher education audiences.

(7) *The publication of a Newsletter.* SMSG has attempted to keep the education community aware of the purposes, developments, and findings resulting from use of the materials.

(8) *The publication of an Abstracts journal.* SMSG is beginning publication of an in-house journal containing abstracts of research reports. The purpose is to make current research findings readily available to SMSG developers.

(9) *A group of books designed for teachers and advanced secondary school students dealing with special topics* (published by Random House).

(10) *A four-year longitudinal study, to determine how children learn mathematics.* This takes another look at the elementary school program, involving research on a smaller scale before going into more large scale curriculum development. From tests given at beginning kindergarten, ending kindergarten, first, second, and third grade levels, some differences were noted in rate, amount of learning, and heterogeneity of regression. Half of the groups were from low and half from middle socioeconomic levels; half used the state series of textbooks, while half used SMSG.

A complete list of the available published SMSG materials is available from Vroman's.

INTERVIEW with E. G. BEGLE, Director
SCHOOL MATHEMATICS STUDY GROUP

C. Alan Riedesel and Marilyn N. Suydam

(4 February 1969)

Q. *What are the purposes and objectives of the project? I think these are probably familiar to most people in mathematics education, but perhaps not as familiar to some of the audience that we have.*

A. Well, one of the basic objectives of S.M.S.G. is to help provide for students in school a mathematics program which is more appropriate than the programs that were available in the past for the kind of world they're going to live in when they finish school. In order to do this, we conduct research and development in mathematics education. Sometimes we spend most of our time in development, sometimes most of it on research, but we feel that we have to do both. Basically, as I say, it's to try to get the pre-college mathematics program modified in such a way that it becomes more useful to these kids, taking into account the kinds of uses they may have to make of mathematics when they finish school.

Q. *How about the instructional approach or the methods or the philosophical basis for the teaching approach throughout S.M.S.G.?*

A. Now here I think we differ from some of the other curriculum projects by not having a fixed position on pedagogical procedures. We have tried to write our materials in such a way that they can be used in whatever way the individual teacher finds most comfortable. We do, however, clearly feel -- and this shows up in any one of the teachers' manuals you want to look at -- manuals you want to look at -- that learning mathematics cannot be a passive process, that the student must be actively involved in the process. But aside from that we don't have any specific philosophical approach or any specific pedagogical procedure that we are advocating.

Q. *I've heard people say that the teaching approach is eclectic and I would guess that you'd agree.*

A. We tried to write our materials that way.

Q. *Let's say a student went through twelve years of S.M.S.G. What would he be prepared for when he finishes high school?*

A. Well, it depends on how able the student is. Because how fast he can go through the program depends upon his ability. The very able student should be able to handle, while still in high school, a full year of calculus as it is now taught in the better universities. A somewhat less able student, taking what you would usually take under the normal college prep program, should have all of the prerequisites for a rather sophisticated calculus course as his first course in college. On the other hand, we feel that there are plenty of students who, instead of following that pre-calculus program entirely, can branch off at various places and add some other interesting and useful mathematics. We have, for example, a senior level course on computing which a lot of students will find useful. There are other possibilities. There's linear algebra that can be done in high school, or probability. We don't have any single unique program.

Q. *Getting back to the elementary level: I've heard speakers say on occasion that programs such as S.M.S.G. are designed for the top twenty percent of the elementary school population. What's your reaction to such a comment?*

A. Well, that is clearly false. It's designed for the average student. Whether the design was carried out successfully is another matter. But they were deliberately written for the average student, just as our high school materials were written for the average student taking the college preparatory program. I think to say the top twenty percent only -- I've heard more extreme statements such as the top two percent -- such a valuation is obviously not true.

Q. *Would you say that it's your feeling that the majority of elementary school students could handle S.M.S.G. if it was paced at their particular rate -- some finishing early and some finishing a year late?*

A. We certainly have no evidence to contradict that. We are building up a lot of intuitive faith that the answer is yes. This would also mean that a considerable number of elementary school students could finish the normal elementary program before they now do.

Q. *What type of things do you do to evaluate the effectiveness of S.M.S.G.?*

A. Well, we do two things. The first one I think is terribly important although it's not a very objective sort of thing. Every time we decide that another kind of textbook needs to be written, we first of all involve classroom teachers in the writing at the very beginning. After the preliminary version is done, we send it out to a substantial number of classrooms. The sort of feedback we want from the teachers is not 'the students did two points better on such and such a test'. The feedback we want is 'how teachable did you find this material? What parts were hard to teach? What suggestions do you have for improving it? What parts went well? What further information do you need in the teachers' commentary?' and so on. In other words we want to make sure, first of all, that the reasonably capable teacher, with a small amount of in-service assistance, can handle the material. So that's step one.

Step two is to find out in some detail what happens to students. We did early in this decade some rather small one-year studies -- enough to convince us that we were on the right track. In 1962, I think it was, we started a very large five-year longitudinal study, which included students starting at fourth grade, seventh grade, and tenth grade in a variety of mathematics programs. These students were tested rather extensively twice a year for the next five years. Tenth graders, of course, were only tested for three years, then they left high school. We have included in the test batteries not only mathematics scales but also a number of psychological scales, both cognitive and affective. We've collected a good deal of information about the schools, about the communities served by the schools, a little bit of socioeconomic information about the students, a good deal of information about the teachers, fairly complete information about the textbooks that the students used each year, and so on. We're now in the process of trying to extract useful information from this vast amount of data. We're not really trying to evaluate ourselves as much as we're trying to uncover information that can be used for further improvement -- not just by us but by everybody concerned with mathematics education.

Q. *Do you see any particular dangers in the 'uninitiated' using your material?*

A: Oh yes, this can happen. It happens when a teacher, or more often a school administrator, decides that the material is going to be used and does not provide the classroom teachers with some guidance and hopefully some in-service assistance. A teacher left to herself without this guidance finds it rather easy, I think, to misconstrue the objectives, to pick out certain particular aspects and overemphasize those, and just move off in a tangent. That could happen very easily. We have therefore, from time to time at the very beginning, encouraged institutes and careful study of the purposes of the materials.

Q: *What type of activities do you engage in for in-service education for teachers?*

A: We do research and development, but we do not do the next step which is dissemination or implementation. We have prepared a number of books that can be used in in-service programs but we do not, ourselves, run any such programs.

Q: *I would guess that large numbers of N.S.F. institutes and the like have used your in-service materials?*

A: That is correct. And I think in more recent years a fairly substantial number of school systems have realized the necessity of an appropriate in-service program, not just when they take up our materials but when they make any major change. I think the large school systems are becoming a good deal more sophisticated about that.

Q: *Should math and science be integrated as far as the elementary curriculum is concerned?*

A: No, they shouldn't. In fact they can't be, because they have different objectives in mind, they develop ideas in different sequences, and any attempt to integrate inevitably means subordinating one to the other. I don't think it will work. On the other hand, I think there are many places where they can be coordinated and that one could be used to reinforce the other and we should clearly take advantage of such cases whenever we can. But to ask for real integration, I think, is asking for the impossible.

Q: *Is this a matter, though, of the way in which the science materials are written today, and could you rewrite them more from a mathematical point of view?*

A: I think, yes, this could certainly be done but I think this would be subordinating the science to the mathematics and I wouldn't have the presumption to think of doing that. I think there are important scientific ideas that I just wouldn't recognize and in an attempt to use science to support the mathematics program I could do great injustice to the science program.

Q: *So when you say coordination, what kind of thing do you specifically* *mean: that you'd just be using the science as a setting for mathematics?*

A: There are some of the activities that go on in some of the science units that have been written for the elementary school which could be picked up and followed by the mathematics part of the program. The science could be used to introduce some mathematical ideas, but you can't expect that every mathematical idea is going to be introduced by an appropriate piece of science. Whenever it can occur, fine, and vice versa -- whenever some mathematics can be used to support and improve a scientific idea, let's do it. But you can't expect that every bit of science is going to be tied in with mathematics.

Q: *Let's go back to the question on social math. This has been one of the reactions to S.M.S.G.: that social applications are missing. How important is it?*

A: You'll have to explain what you mean by social math.

Q: *You know over a period of time during the 1930's when people such as Guy Wilson, Leo Breuckner, and Grossnickle, were writing very heavily toward the social needs of mathematics -- 'grocery store arithmetic' -- a chief criterion was that everything had to be tied to every-day life for mathematics. It seems to me that in the era of the fifties and early sixties, some material was written very little in the way of application. The applications came at the end -- very little was done with applications during the ongoing part of the program. Current literature tends to push a combination of applications which might replace 'pure' mathematics. What's your feeling concerning how much stress should be put on application?*

A: Well, that's difficult. I certainly would be very unhappy if children ended up, say at the end of junior high school when some of them can now quit taking mathematics if they want, I would be very unhappy if they ended up feeling that mathematics was just a game, that it wasn't related to anything real. I'd certainly want them to feel that mathematics is *a* way of looking at the world we live in, and of helping us to understand it, to analyze it, and to solve problems that the world poses. On the other hand, I don't think it's necessary, in order to get this understanding of the role of mathematics, to tie each individual piece of mathematics into a practical application. In fact, it's impossible, because much of the mathematics we teach, while it does have applications, is taught at a time when the student doesn't understand that part of the world to which it can be applied. We have to teach mathematics in a certain order because that's the way mathematics is. You can't do anything with fractions until they know about whole numbers, and so on. It just happens that lots of mathematical topics are taught at a time when the students do not yet have *any* understanding of that part of the world where it can be applied. So you just can't tie them immediately to applications and you can't use *those* applications to motivate them. I think trying to tie everything in mathematics immediately to an application is impossible, but there has to be enough attention paid to the uses of mathematics to make sure that students really understand that that's why they're learning mathematics. It's not just a game.

Q: *How do you do it?*

A: Well, you do have to put in applications whenever you can. The problem is to find the right ones.

Q: *Let's say that we have a portion of elementary school mathematics that has a number of very good science applications that would make good sense to the youngsters. Where in the sequence of teaching the concept would you put the applications? Obviously we could include them all the way along or we could start out with the application and move to the mathematics, or you could start out with* *the mathematics and then move to the application. What would be your preference?*

A: You're posing some pedagogical questions for which I don't think we have any answers but are certainly worth looking into. I think my own feeling is that any new mathematical idea should be drawn out from ideas that are already sufficiently familiar and concrete to the student so that you're moving from the concrete to the abstract. I don't think, until you get about to the graduate school level, can you go the other way. Certainly not in the primary and intermediate school. You must go from the familiar, the concrete, to the abstract. So that if you have a real-world situation that children already understand -- I don't think they understand, when they're in sixth grade, much about mortgages and insurance and that sort of thing -- but if you have something that they do understand, that's familiar to them, and if it can be used as a vehicle, as a source from which a mathematical idea can be drawn, then that's fine. But if you can't find such a situation, then I don't think you can do it.

Q: *You used the word motivation -- what provides the motivation?*

A: If I used the word, I'm sorry. No, I shouldn't have said motivation because I don't know what motivation is all about, myself, but the emphasis should be on developing mathematical ideas and goals in what the students are already familiar with -- start with the *more* concrete and then abstract the new ideas. It's very important to notice that once you've abstracted a new idea and worked with it long enough, then *it* becomes familiar to the student and becomes the concrete on which later abstractions can be developed.

Q: *I think this is when you get into some difficulty too, where people won't build on a mathematical idea, they want to then go back and build on the original, concrete idea which the youngsters have already bypassed to get at the mathematical one...*

(Interview with E. G. Begle -- C. Alan Riedesel and Marilyn N. Suydam)

You're involved in something related to what Piaget is talking about in terms of his levels?

A: I wish I understood Piaget well enough to be able to answer that question.

Q: *There seems to be real misunderstanding of the word 'concrete'. His 'concrete stage of learning' is not what we commonly have meant in arithmetic by 'concrete'.*

A: Well, he seems to have just these three major levels, and I think of mathematics as consisting of many, many, many different levels.

Q: *Go back to "motivation", in quotes -- in your work with the junior high, why did the students learn better under a given program? They hadn't learned before; they did learn under this program. Obviously something related to "motivation" was involved there.*

A: Well, I wish I knew exactly what made things work in the experiment. I suspect that one of the most important things was that they were given a chance to succeed. They were given tasks that they could actually carry out and that was something new for them as far as mathematics went. Once this was possible, then I think that a fair number of them were perfectly willing to go along and continue through the schooling process and find it non-objectionable. But I suspect that the main thing there was that they were given a chance to succeed.

Q: *Then rather than some specific goal of introducing it, or something innate in it, that feeling of success with it was more important than a relational motivation.*

You said earlier that on the basis of your slow learners study you'd be willing to generalize and you got as far as saying that this is the kind of group that should be set up -- homogeneous and on the basis of I.Q. and initial mastery -- any other generalizations there?

A: Well, that study was done with below average students. I would be willing to generalize that to the entire spectrum of the junior high school. I think that the above average students could profitably be grouped. Incidentally there is a little research support for this -- a study done by Passow and Goldman on accelerated students. That also indicated that moving the brighter kids along faster was a better way of handling it -- once again, adjusting the speed to the students in a capacity. The other thing that I would add is, you would not just adjust to that but to where they start at the beginning of the year.

Q: *Now, if you extrapolated that down in the elementary school, where would you start this type of differentiation in terms of moving some along more rapidly than others?*

A: Well, I would suspect that the place to at least begin to look into this would be around grade 4. It may be that even that's a little too late -- I don't know. But I suspect that just about grade 4 is where we ought to be looking next.

Q: *I'd have said offhand to start at grade 3 but I don't know.*

A: I don't know. It would depend on the individual child.

Q: *Some research indicates that by first grade, you've already got a wide range.*

A: Oh yes, but I'm not sure that it's necessary to make life that complicated that early.

Q: *While we're talking about individual differences, would you very quickly go over your feeling on individually prescribed instruction and on math labs?*

A: Well, if it is accepted, if we get more research support for this belief, there it the possibility that the length of time that a student devotes to studying a particular topic should not be the same for all students. Then you have to ask, how can one handle this in the classroom in the school. One suggestion that is normally made is individually prescribed instruction and this may indeed be the way to do it. I would like to have a good deal more evidence for this, however, because my feeling, from talking to any number of teachers and observing any number of classes, and so on, is that the learning of mathematical ideas must involve active participation on the part of the student. When I see that sort of active participation -- incidentally, I don't mean discovery learning by this, I mean active participation -- I see this going on, usually in some sort of a social context where there is interaction among the students, and back and forth between the teacher and the students. With I.P.I. there is much less of this, so I would be very curious as to how well I.P.I. can handle things.

Now the math lab encourages a great deal of activity on the part of the students, which I think is fine. My concern here would be in whether there is enough direction supplied to the activities. Mathematics, whether you like it or not, is terribly sequential. If things are omitted, then later on there is trouble.

Q: *What would be your feeling of blending these two or three approaches? It seems like people always take the view 'I'm going to use I.P.I.' or 'I'm not going to I.P.I. but I'll put everybody in the same place in the textbook'. Do you think it's possible to blend individually prescribed instruction, with a sequence such as S.M.S.G., with mathematics laboratory material, at the right time, on the right concept?*

A: It may very well be. I'd love to see some work done exploring the facts of different blends of these.

Q: *I suspect that a mix is what we're looking for and most people don't experiment with mixes for some reason or other.*

Project Title: Individually Prescribed Instruction Project
Location: Pittsburgh and Philadelphia, Pennsylvania

Background

The Individually Prescribed Instruction Project (IPI) originated at the Learning Research and Development Center of the University of Pittsburgh in 1964. The project began as a cooperative venture at the Oakleaf Elementary School with the Baldwin-Whitehall School District in suburban Pittsburgh. The intention was to establish a working model in which the school curriculum was individualized. The system requires the selection of method and mode for each child; he progresses at his own rate, with mastery as his goal.

In the mathematics curriculum, placement tests are administered on thirteen topics before instruction. From the resulting profile, both long-term (six weeks) and shorter (daily to weekly) prescriptions are written. The materials are divided into almost 500 skill booklets, each to help the child reach specified behavioral objectives. Pretests are administered; as the child completes a section, he takes a curriculum embedded test, additional similar instruction may be prescribed, or the posttest is taken. Generally the mastery level is eighty per cent. The system is a guide, and teacher judgment can always override the system. Plans are underway for the development of a computer management system to aid the teacher by resolving clerical problems in the effective use of the materials.

Since 1966, Research for Better Schools, one of the regional laboratories sponsored by the U.S. Office of Education, has cooperatively joined the Learning Research and Development Center in the development and demonstration of the IPI project. The LRDC is now primarily oriented toward learning theory aspects of the project, while RBS is interested in diffusion. Their goal is to establish totally individualized schools, from pre-school through adult education. During 1968-69, RBS was working with approximately 100 schools using IPI; about 100 more will be added for 1969-70. There are six demonstration schools, selected for varying characteristics such as suburban, farm, and inner city. In less controlled situations about 23,000 children and 850 teachers are involved.

INTERVIEW with ROBERT SCANLON

RESEARCH FOR BETTER SCHOOLS

Marilyn N. Suydam
(10 March 1969)

Q: *What are the purposes and objectives of the project, both now and for the next few years?*

A: As you know, Individually Prescribed Instruction has several basic objectives. They are as follows:

1. To enable each pupil to work at his own rate through units of study in a learning sequence.
2. To develop in each pupil a demonstrable degree of mastery.
3. To develop self-initiation and self-direction of learning.
4. To foster the development of problem-solving through processes.
5. To encourage self-evaluation and motivation for learning.

With the math program for the last four years, the primary objective has been to individualize what *is* during the next three years. The Learning Research and Development Center intends to take major leadership in rewriting the content of the math program.

Q: *What is the philosophical basis for the teaching approach?*

A: The philosophical approach is based on experiments conducted by Bob Glaser dealing with programmed instruction. The approach in IPI is really an application of the principles of programmed instruction to Individually Prescribed Instruction. This system differs from most instructional systems along the following dimensions:

1. Detailed specifications of educational objectives.
2. Organization of methods and materials to attain these objectives.
3. Careful determination of each pupil's present competence in a given subject.
4. Individual daily evaluation and guidance of each pupil.

5. Provision for frequent monitoring of student performance, in order to inform both the pupil and the teacher of progress toward an objective.
6. Continual evaluation and strengthening of the curriculum and instructional procedures.

Q: *What methods of instruction have been used on the project?*

A: As you know, Individually Prescribed Instruction is an attempt to change the role of administrator, teacher, and pupil in the program. The teachers are responsible for instruction. The attempt is to have teachers apply principles of a diagnostician and prescribe both instructional materials and settings for the youngsters.

Q: *In what ways are individual differences considered?*

A: The primary consideration is through rate of learning. This was one of the initial attempts and initial work was to be sure that different kinds of materials are available for different kinds of youngsters that take care of their own individual rate. Much work is now proceeding in the use of instructional conditions. That is, prescribing the proper mode of instruction as well as different kinds of materials for different kinds of youngsters.

Q: *What are the major objectives that a student should have reached when he completes the program?*

A: Individually Prescribed Instruction in mathematics is used with almost every conceivable kind of audience in the research stage. The major objectives, to a large extent, are to have the youngsters work independently using the proper kinds of materials to achieve mastery of basic mathematic skills (and attain literacy in mathematics).

Q: *What type of activities do you think are necessary for in-service training for the teachers involved in the project?*

A: The major approach in trying to retrain teachers is to develop a system that will permit the individualization of the retraining of teachers about individualization. Several dimensions are necessary here. The first is providing enough background information and specifics about the mechanics of the system so that teachers will feel comfortable in the mechanics of the system itself. Secondly, attempts are being made to build in continuous training so teachers begin to analyze the data about youngsters as well as the data about their own prescriptions to make better decisions. The training program also involves the retraining of the administrator, and the assumption is made that the administrator can assume major responsibility for retraining his own staff.

Q: *How is the project being or to be evaluated?*

A: Research for Better Schools has assumed the responsibility of continuing to ask, 'How good is it?' and, 'Where can it be improved?'. The 'Where can it be improved?' data are specifically information being gathered about the success the kids are having in the program or the lack of success and a hard look at the kinds of materials that are used or the kinds of settings that are used. This information is translated to the University of Pittsburgh for further refinement of the project itself. 'How good is it?,' you know, is another serious question. Of the last two years, we have tried to use pretty much standard techniques in asking, 'How good is it?' to give us license, so to speak, to develop our own techniques. There are major problems as you well know in using standardized instruments in measuring achievement, and the lack of sophistication in attitude instruments. So we are in the process of developing some of our own.

Some major research documents are also now being prepared jointly by the Learning Research and Development Center and Research for Better Schools and should be released in the near future.

Q: *What do you picture as being the major impact of the project?*

A: It seems to me that the goals and elements of IPI when demonstrated are quite visual to people who see it. It has captured the imagination of a large proportion of the school community. This is evidenced by the thousands of visitors that visit the demonstration schools and the requests that are answered daily here at RBS. For example, we have about 300 letters a week asking for information about IPI and every major educational journal as well as many non-educational journals have carried stories about the project.

Q: *Do you see any potential dangers of improper use or interpretation of materials or ideas from the project? (Are there safeguards you'd like to suggest?)*

A: There are more than safeguards that I would like to suggest. For example, schools must go through a rigorous application to apply for participation in the project. Only schools meeting the criteria are permitted to use the system and this will be true at least until it's commercially available. The danger I see is once the project is commercially available. Commercial companies may see IPI as an instructional system and not necessarily as a set of materials. I hope that the commercial people will be able to maintain the same kind of safeguards.

Q: *What are the major plans for the future?*

A: Research for Better Schools would like, by 1972, to have developed instructional systems for the total elementary school so we can demonstrate individualization of all areas. Pressures are mounting to expand the project beyond the elementary grades, work must be begun soon on the secondary level. It seems to me that the principles are equally applicable to adult education and could have a major impact on vocational technical education. Once funds are available, we would like to move full-steam ahead in all of these areas.

It's very important that math be related to real-life situations. 'What's the real need?' and 'How can you apply this?' are important questions.

Q: *Should mathematics and science be integrated?*

A: Yes, to the extent that the skills in one area are needed for competency in another. Some work is now being done with the science-math projects in the IPI program. The data from science units are needed by the teacher to help her proceed in math. I think Warren Shepler pointed out some of the areas, such as volume, measurement, and area that are related. The child doesn't have to go through the same material in both areas: we extend and reinforce from one area to another. There is increasing blending or integration.

Q: *What materials have already been produced?*

A: A whole bank of materials in math, reading, science, spelling, and handwriting have been produced which include diagnostic tests: specific curriculum materials related to objectives, as well as teacher training materials.

Project Title: The Madison Project
Location: Syracuse, New York
Director: Robert B. Davis

Background

The Madison Project was originated in 1957, primarily as a vehicle for exploring ways of revitalizing the teacher education program. The intent was to develop a closer relationship between what was being taught in mathematics and mathematics education courses, and what was going on in the schools. In the course of the exploration to meet this need, the development of materials which could be used effectively with children to promote the interaction objectives evolved. The kinds of creative learning experiences which children can have in and out of school became a focal point for consideration. Thus, the Project is adapted both to enriching the classroom experience for the child and helping the teacher plan experiences which will involve the child both cognitively and affectively.

The materials which have been produced for children are designed to provide worthwhile, interesting supplementary experiences. For both teachers and children, use of a mathematics laboratory situation is encouraged. Currently Professor Davis is conducting both pre-service and in-service courses which are highly individualized. Pre-service teachers spend at least some hours each week in the schools, actually working with individual children. The problems which they meet are then discussed in class, with everyone participating in an analysis of problems which have been met and suggesting possible solutions. In-service teachers are free to select from a wide variety of materials and experiences, with a great deal of interaction.

Perhaps a summary of the mode of operation in the Project is implicit from Davis' words: 'Guess--try--watch what happens--learn what to do next.'

INTERVIEW with ROBERT DAVIS, Director

THE MADISON PROJECT

C. Alan Riedesel and Marilyn N. Suydam

(10 March 1969)

Q: *What was the origin of the Madison Project?*

A: Well, the actual origin of that was that various faculty members in the mathematics department at the University were working with undergraduates who were going to be teachers, or doing in-service courses for teachers, and the undergraduates or the teachers were very skeptical as to whether the faculty knew what they were talking about. They would say, 'Well, that may be a great idea for your university students, but it would never work with my class.'

So we began our work in schools somewhat imperceptibly (which, incidentally, is one of the reasons for the ambiguity concerning the date when we started). It was really simply a question of the teachers saying, 'Look, you guys are all college professors. You've never seen what a sixth grade class in my school is like.' And so various of us 'college professors' started taking classes in nearby schools, and pretty soon we were getting surprisingly good results with allegedly 'hopeless' children. You have to be careful how you say this, because really it sounds awfully arrogant and I don't feel that way about it at all. I think anybody with a lot of the outside freedom that we had could look at various class situations and say, 'Gee, this teacher has got the case rigged against her so she can't win.' Well, it wasn't rigged that much against an outsider. We would just come in and not feel as bound by the curriculum, not feel as bound by the mores of the school, and so on. Then also, since there were more people coming into the school, you could have smaller classes and special things like that.

Before long we got to the point where we could get quite noticeably better results than the school had originally been getting. The consequence of this was a change in people's attitude toward what we were doing. Instead of saying, 'You could never do that,' they wanted to come visit and see how we did do it. Really within that first year the emphasis switched from people saying, 'Gee, that would never work' to people saying, 'Hey, I've been hearing about what you're doing over at Madison School and I want to come and see it. Can I see it sometime?' We'd say, 'Okay, just come.'

(Interview with R. Davis -- C. A. Riedesel and M. N. Suydam)

Q: *What's the ultimate scope of the project?*

A: To my knowledge, this was perhaps the first project that was concerned with the mathematical content and with everything about the school atmosphere simultaneously. And so, I think of our scope much more in terms of what we'd like to see school be like. That's really why we stuck with the name, the Madison Project. We didn't want to use the word arithmetic or mathematics or anything like t that because potentially we could be dealing with any area. For instance, one unit that I played around with a little bit--it's not one of my triumphs by any means--is a stock market unit. It seems to me potentially one could use looking at the *New York Times* or the *Wall Street Journal* every day as a way to get all kinds of things taught. You could include the meaning of the peace talks in Paris, the discovery of oil on the northern shore of Alaska, where is the northern shore of Alaska, what kind of problems do you have getting the oil out, which are the companies that have found oil there, what was that problem on the Santa Barbara coast out in California, which oil companies were involved, where is Santa Barbara anyhow, what do you do when you get an oil leak like that, why are they looking for oil underneath the water, where else are they looking for oil underneath the water.

Obviously, out of what could be originally a unit on graphs or fractions or percent, you could ultimately pull all the rest of these subjects into the discussion. If American Airlines which sells around 30 goes up three points, and I.B.M. which sells around 300 goes up three points, what difference does it make? Now that's obviously a question in ratio, proportion, and percent. But you immediately get into these other questions. So really, we've deliberately not delimited the scope, because we see so many possibilities on the horizon.

Q: *Many programs in 'new mathematics' were very much devoid of applications in terms of social situations for everyday life. It seems to me there's a little bit of a swing toward applications now. Would you sense this as being a definite movement?*

A: Well, that's one that I think you have to be careful of, because applications come in in so many different ways. One of the questions is, 'Should you teach theory first and then apply it?' Well certainly, that's often done. In the days when I was at M.I.T. that was the prevailing approach there. First you taught the theory of whatever it was, and then you applied it. But M.I.T., at the university level, seems to me to be swinging somewhat in another direction now, and increasingly they put a student in a lab with minimal previous instruction and tell him, 'Do something interesting,' which seems to me, if you can carry it off, much closer to what really happens in adult professional life. We know in the past how few graduate students in mathematics ultimately went on to become productive adult mathematicians. What the world says to adult professionals is, 'Do something interesting.' Within mathematics education, it doesn't say 'You shall work on the fourth grade,' or 'You shall work on math for the preschool child' or anything like that. It says, 'Do something worthwhile.'

Ostensibly, that's what we say to kids when they get to about the doctoral level. My impression is that M.I.T. is increasingly trying to say that even to freshmen. They'll have more and more lab situations where there's just a lab there and they say, 'Go in and perform an interesting experiment.' I find myself that I learn much more in the course of doing this.[1] This question of *why* you do things seems to me to loom larger all the time, all the way from elementary school through adulthood. We don't just 'learn' or 'perceive.' We perceive what is relevant to our personal internal purposes. I really am getting sold on this bit about perception as answering questions. If I have lots of questions, I think, 'Gee, there's some interesting stuff in genetics'; the first thing I've got to find out is who's working in genetics and what do people know, and what kind of methods are they using, and so on. But that's very different from what happens if you give me an assignment to read a book on genetics. Because then I don't *really* have any questions, not honestly. The British make a big point about this. Are you answering one of your *own* questions or are you answering one of somebody else's questions? Because they claim you perceive it differently. I think that could be true.

So for our present use of applications I don't think that what we are doing nowadays is necessarily all that different from what we did a few years ago with theory. Some people have criticized our approach, but I think basically we like to confront people with confusion and leave them with the job of straightening it out. That's roughly the same whether the confusion is lots of formulas that they've got to straighten out, or whether the confusion comes from physical or social situations. You might be trying to select an appropriate set of axioms for a sort of 'traditional algebra'—we did that 12 years ago with fifth grade children and they selected a perfectly satisfactory set of axioms, so that they were able to organize the algebraic chaos. But it's not that different with the game of Nim, or with the Tower of Hanoi puzzle, or with the breaking strength of yarn, or with a computer language like LOGO.

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[1] Incidentally, an excellent article has just appeared on this question: Keith Hirst and Norman Biggs, "Undergraduate Projects in Mathematics," *Educational Studies in Mathematics*, vol. 1, no. 3 (January, 1969), pp. 252-261.

As a matter of fact, that's what I like about that computer terminal we've got; really, the most exciting thing is to make it *work*. Since Bolt, Beranek and Newman are in the process of writing the software right now, there is no manual for LOGO. Nobody outside of Bolt, Beranek and Newman really knows all the possibilities--if you want to find out, you've got to make a good guess and try it, and if it doesn't work, make a good guess on why it didn't, and try again. You just can't look in the manual--there isn't any. So I find that really quite exciting. Of course, I admit that some people don't like it at all. I guess that's life.

One of the things the Cambridge Conference has argued for is the possibility of applying math to other math. Now I think a few people overdid that argument because they were prepared to say that if you don't do any applications at all except to math, that should be satisfactory. I don't think it would be satisfactory for some students. Some children (and for that matter, some grown-ups) care a great deal about the relevance of their work.

In defense of the notion of starting with physical or social situations, I think you could say that anytime you take the reality environment and try somehow to make sense out of it, you're probably doing the right activity. Obviously, part of this will be sort of an application and part of it really is theory development.

Q: *Then you would start with application first?*

A: We would tend to start with application first. Sometimes this would mean 'applying math to math,' but usually it would begin with a physical or social reality.

Q: *What do you think about the integration of science and mathematics?*

A: In fact, that was one thing that we always had in mind. That was the original reason why we chose the name Madison Project, because we always hoped we'd get science in there, and we didn't want to call ourselves a 'math' project. One of the reasons that we've moved so slowly in this direction has been my fear that, since we were mainly mathematicians, we might end up with a mathematician's view of science. That is valid for some people, but a Fermi or a Zacharias has something else: a kind of 'physicist's' view of the world. We would not be doing a good job of relating the two subjects if we presented only the mathematician's view of physics. We were always looking for mathematical properties of science, whereas frequently that isn't what a scientist in that field would be looking for. He might *use* mathematics, but he might think in the imagery of physics or chemistry.

This showed up, for example, in our units on functions. We had functional relationships between all kinds of things. But that's not always what you want. I've watched some elegant arguments by physicists that don't do this kind of thing. I've seen Bill Walton, who was then at Webster College, do some *beautiful* things in physics by using quite different types of arguments--things mathematicians wouldn't ordinarily do. Once he used an invariance argument that was really quite elegant, and was exactly the right tool for the problem at hand. Now of course, obviously, invariance is part of mathematics also, but mathematicians don't always think to use it when they're doing science. They more commonly will measure things and get functions, and get functional variations, and so on. Bill Walton didn't do that. He used essentially a conservation argument, that this has to be conserved and this has to be conserved and therefore this third quantity has to be because I can define it in terms of the first two. It seemed to me very much a physicist's argument. Perhaps the real difference was the way that Bill Walton appeared to think in terms of the models of physics, whereas mathematicians I know, even if they *had* used invariance, would have been thinking in terms of the models of mathematics, although they wouldn't describe it this way themselves.

So I've just been worried that since most of the people that I work with are mathematicians or quite mathematical physicists, we'd end up by distorting our physical science. I don't worry about that so much on the stock market unit at this point, because we're not making any pretense to be teaching economics or business or anything of that sort, but really just trying to develop something more interesting than the geography of Alaska the way I studied it when I was in school. I'm sure we can improve on that! If you really and truly wanted to teach something about economics, then I'd probably have the same feeling that we may not be seeing things as economic questions, but merely as mathematical questions, and there probably really is a difference.

Q: *If things went according to what you would like to see, what would be the next major tack of the Madison Project? In other words, where would you be going in the next three years?*

A: Obviously that's really one of the very important questions. Unfortunately your plans depend upon your expectations. You're always planning--unless you're just psychotic--against some reality expectations. If we change our expectations we have to change our plans. The thing I'd like to see happen would be to get lots of experimental schools run by different people on different philosophies. Now you've got to decide, 'Okay, is that a realistic possibility or not?' It's very hard at the moment but it looks like it might happen. Right here in Ithaca there's a school that's scheduled to be closed. All the teachers in it have been transferred out to other schools and the building is to be unoccupied in September. Now one of the parents' groups wants to take it over and run it as an experimental school. They're getting quite a bit of interest and support in the community, and they'd really be starting from scratch. There would be no faculty there, no anything. They could go a long way towards building the kind of school they really want.

I've always liked David Hawkins' remark that the problem with the 'independent variables' in education is that actually they're constants. I think the thing that hits you as you look at different schools around the United States is that they really are all pretty much alike. There *are* differences, but mostly not very big ones.

I'd like to see *every* serious philosophy of education tried out. A particular school would select its own philosophy, develop it for all they were worth, and have the courage of their convictions when it came to implementing it. That would mean we would have many different types of schools, some quite unlike others. Then I would let the *customers* choose, as they can in buying a car or choosing a restaurant.

You name almost any theory of education you want and I'd like to see some people really go at that hammer and tongs. That's why I like the school with no building in Philadelphia. I'd like schools where attendance is voluntary. I'd like some farm schools where you could go horseback riding and do stuff like that and again maybe attendance in class was voluntary. Much of our work where we use kids is often after school, or Saturdays, or summers. That's voluntary and we've followed some of these kids for as long as five years. I think that gives you a measure of what's happening. Will the kids come on Saturdays for five years? I even like this approach in my workshop for teachers here at Ithaca. I can't tell whether these Monday evenings are really worth anything to them, but one of my criteria is going to be, will they keep coming, because really they're not getting any serious credit for it. They're mostly on the top salary level. They're not degree candidates or anything. So, will they keep coming? That gives you a lot of insight into what you're offering them.

But the key to all of these different kinds of schools would be *diversity* and *consumer choice*. I don't rule out things like the stereotypes of the Bereiter-Engleman approach where allegedly you have a sort of controlled, benevolent fascism or whatever you want to call it, where you really tell kids what they've got to do and you make them do it. My feeling is we just can't study these things right now because you don't find any real variations, or not very much. Every school has a little bit of this and a little bit of that. That's one reason that I've made judgments in things I've written, that I've admired Summerhill as much as I have. I admire Phil Pappas' Warrendale School outside of Pittsburgh. They really took their philosophy and carried it out very far. That's what I'm trying to do with this Cornell course. I am not giving these teachers assignments. I just sort of sit around and when someone begins to look like he wants to do something, he can. I help as much as I am able. I'm not at all convinced that this is a good way for me to treat these teachers, but I see some virtues in sticking to your guns and going down, if you do, with all flags flying. It seems to me that this is one way to get the teachers to rethink what this is all about, and perhaps to make changes in their own teaching. *That* is what counts--not what they do for *me* in the workshop.

I think the thing that would impress me the most would be to get many *different* kinds of schools, and then I think you'd find you could get very useful research, because the obvious guess is that you'd then be able to tell of a certain teacher, he will make it in this kind of a school and not in this, or a certain kid, he will make it in this kind of school but not in this. You know, this keeps *almost* happening, but it's hard to pull it off, and we keep slipping back into 'creeping lowest-common-denominatorism.'

Q: *Would you try to formally teach the same math program in all of the different patterns of schools?*

A: When you ask it that way I come back to my original guess that the curriculum and the whole environment are sort of inseparable. For example, one school I'd love to see is the 'mini-school' that Paul Goodman has written about. In that kind of school, children and adults would come in in the morning, and one of the first things they might do would be to make breakfast together. As the parent group here at Ithaca has pointed out, you can arrange the economics to get much smaller class sizes without pushing up the expenditure if you do it right. For example, don't require college graduates for some of these jobs. Making breakfast doesn't require a college graduate! Of course the children would help. My own daughter learned things like 'half a cup of flour' this way. If the recipe called for it, she could make pancakes, and things like that, and she knew half a cup of flour or a cup and a half of flour. She didn't learn abstractly one half or one and a half. The first thing she did was make pancakes and go from there. Then in this kind of school, you do things like plan lunch, you go out and buy lunch, and you make lunch. Obviously, you accommodate as many activities like this as you can.

Now that's going to *have* to be a different math program from one that you're doing in an Engleman-Bereiter setup. I think they ought both to go side by side. I don't see, short of running these things for 20 or 30 years, that we're honestly going to know what's the difference. Because I'll argue that if you really want to know what educational failures are like, we can see them, but we've got to know what they look like. I would not be interested in mathematical education if I weren't interested in education generally. I don't think mathematics by itself is honestly that important. Of all the crises that may overtake us in the United States, and there certainly are plenty, I see none of them being primarily mathematical.

I like some of the things Friedenberg has said about kids. I think one of the kids who is a failure is the kid who sort of 'opted out'--I don't mean so much by taking L.S.D., but he 'just goes along with things.' He'll get a job and do what they say he's got to do. And if he gets fired, okay he gets fired. But he just goes along with things. It intrigued me that Jacqueline Grennan, who hadn't at that time read Friedenberg's writings, also picked this kind of child as one that she considered a failure. If young people get to the point where they just don't feel strongly about things, they wouldn't particularly fight over anything, they take things as they come, no matter what--why, that has *got* to be a major educational failure. Would people like that oppose an Adolf Hitler--or would they somehow try to get through each day and attempt to overlook the murder of six million Jews? There a horrible example: Nazi Germany was an educational failure. That's the sort of thing that you don't want to have happen, and an educational system somehow should protect you against things of that sort. I think you could find things in our own domestic policies that are really reasonably called educational failures. They *shouldn't* have happened.

I don't know if you ever read a little thing I wrote for the Council for Basic Education. I was attempting to say this there, though I don't think it necessarily came through. But honestly, try one of my tests. Just sit in taxis in New York City and listen to what the driver says. Ask yourself what you think about his education. I don't mean his use of the English language or anything like that, but rather the kind of things he'll say about Negroes, the things he'll say about Puerto Ricans, the things that he'll say about rich people or poor people or people in New Jersey. These are just not attitudes that are compatible with a successful democracy in the long run. I wrote that C.B.E. note before the George Wallace phenomenon, but I think that today you can just read the C.B.E. note and you can say, 'Okay, that predicts the George Wallace phenomenon; we're going to have it because people just don't see anything other than that.' Comparing education is comparing culture. That's why you have to run some of these schools for years, maybe decades, before you really know whether you're producing a desirable culture or not. That's really Alexander Sutherland Neal's argument for Summerhill. In fact, a book I was very much impressed by was Nathaniel Cantor, *The Dynamics of Learning*, and he uses some of the same arguments. He cited, as one example, our prison system as an instance of an educational failure. A *really* responsible citizenry wouldn't put people in prison the way we do today.

It seems to me the whole point of teaching math or anything else would really be to try to aim for a humane society. The *New York Times* has had quite a bit about this. They've said we've created the greatest society in the world but not a great culture, not even really maybe a viable culture. It's interesting to listen to some of the religious programs on the history, for example, of John, and realize that he was a political prisoner, and then match this up against things that you could say about Eldridge Cleaver. It's awfully surprising, I think, to realize what happened to early Christians, the extent to which Christianity was a religion of slaves in a large part. Was St. Francis a Beatnik?

Now that seems to me to be what you've got to play for, that you aim ultimately to get a viable society, that you concern yourself about the quality of life.

We are, in fact, turning in this direction. It shows up certainly with things like the concern for smog--nobody was concerned about smog before. It shows up with the advertisements against smoking cigarettes. M.I.T. recently had a work stoppage for one day to protest military exploitation of science. I think there has got to be more concern about what we're doing all this for. Really school is not merely a place to learn multiplication tables, or how to solve equations, but also a place to look into fundamental questions about myself and my society. School is only *part* of the educational environment--home life, civic life, television, all of the mass media, all of the forms of religious activity play major roles. It seems to me that this whole interrelated system has worked pretty well for us, but we know it isn't perfect by any means. If we don't identify some of its weaknesses and try to correct them, we might lose even the part that does work. After all, I didn't dream Nazi Germany--it really happened.

Q: *But to prepare teachers to teach the comparatively simple things like the multiplication tables, we're having a hard job. How do you prepare them to do this other kind of education?*

A: Of course I agree with that. I think that you've got to worry about all forms of failure. In fact, I worry about this very much now with the tendency to use, as you say, agricultural statistics, or I think even more Department of Defense statistical procedures in education. We could get criteria by which our schools are succeeding but we could turn out a people who are just not a humane people, who are not building a sensible culture. That's one of my objections to this notion that you state your objectives, because people don't usually state as their objective anything real to the effect that we want responsible citizens. Yet above everything else that would be one of my great objectives. Well, they may state something like this, but they're not serious about it.

That's one of the ones I'd be serious about and that's why I say suppose you had different schools. You could begin to find out, okay this school turns out Prussians and basically we don't like Prussians, therefore we don't like that school even if it produces geniuses. This school turns out whatever it turns out and we don't like that kind of person either so we don't like this school no matter what else you could say for it. I don't see how anything under a 20-year experiment with different types of schools could give you much insight into what kind of person will be turned out by that school. I doubt even that 20 years will do it because you're going to have a selective input the way you presently do at Antioch or Bennington or Putney.

Q: *But here's another case of 'are you really going to find out anything?' You're using the word experiment, I think, very loosely there. Go back to an earlier statement where you said, no, you wouldn't teach, for instance, math content the same in all the schools, you'd just let it evolve. Would you really know what you had unless you did some controlling?*

A: I keep coming back to this word 'description.' If you had a good description of what happened in the schools, a reasonable description of what kinds of people went to the school and why, a reasonable description of what kind of people graduated from it and why, you'd begin to have the foundation on which you could build some sort of a theory. This is really how maturity is built, this is how attitudes are built, and obviously this is how knowledge is built.

(Interview with R. Davis — C. A. Riedesel and M. N. Suydam)

Q: *Would you be using sort of a case study approach then to experiment over a period of years, studying on a case study type of basis the various children going through these various types of schools?*

A: I certainly think that could be part of it, but really description can include all of these things. It seems to me it should. How do you describe a city? There are really some nice books that try to describe what Paris or London were like in the 12th century, or the 17th century. What would you do? Well, you want to know where it is, you want to know how many people lived there, you want to know the average age of death, you want to know something about the causes of death. But you also often like to look at it in other ways--suppose you were a peasant, what was it like? Suppose you were a domestic servant, what was it like? Suppose you were royalty, what was it like? You put all of that together. Now that's sort of a description of what it was. All of these add to it. Pictures of the architecture there--what did you see if you lived in Paris in the 12th century? In fact, you can even ask, what did you hear? What noises? What music?

You suggest something I hadn't thought of before. Of course you know Callahan's book, *Education and the Cult of Efficiency*. He argues really that it was the efficiency experts like Taylor at Ford who altered the course of education in the United States (and not entirely for the better either). But lots of people have imposed upon education their own very limited descriptions, despite the fact that we already have better methods for describing such things. We do describe what Paris was like in the 12th century. James Leasor's book *The Plague and the Fire*, Eileen Power's *Medieval People*, and Frederick Lewis Allen's *Only Yesterday* are three excellent examples of effective description. It seems to me that we usually do far less well in education.

If I wanted to describe a school, I'd probably show actual samples of kids' writing. If there was any music program, why not a multi-media approach nowadays? In fact, I even suggest this to you seriously; you might want to think about it some day for *The Arithmetic Teacher*. Why could that not be a multi-media package? But to come back to the school for a second--why on earth not have things like some recordings showing the typical musical performance the kids do? That really will tell you a lot.

I was at a music conference at Yale University some years ago with music educators and two scientists--myself and one physicist, who were really there as sort of observers. I'm just amazed at the fight within music education about 'you've got to give a grade.' Why do you have to give a grade? 'If you don't give a grade you don't have academic respectability.' Now that's a miserable argument. I won't pursue that. That's no argument at all. Some other people would say, 'Well, if you don't give a grade what happens if the kid transfers?' Now the conferees included some very good people from N.Y.U. and from Bennington. One of them said, 'For God's sake, if the kid is majoring in clarinet and he transfers, can't they let him play the clarinet when he arrives at his new school, and listen? Why do they have to have a written document telling them? And if you've got to have a written document, why should it have like a B- on it, why can't it say something like, well, he has a very mellow lower register, he's a little shrill in the upper register, he's quite good at technique except that he has trouble with a C sharp key (if there's a C sharp key on the clarinet which there might or might not be). He's very expressive in romantic music but he can't get the feel for contemporary composers. Now doesn't that say more than a B- does?' Though, as they said, above all else, why not let him take his clarinet along and play it? But you see what I'm trying to say.

Q: *Would you give a brief overview of the patterns of in-service work that you followed on the Madison Project?*

A: All right. Our very earliest start involved just a few people teaching demonstration classes. I don't feel that was terribly effective but it may have been a place to start because it brought some people together. Our first serious venture came when, under N.S.F. sponsorship, we tried what we used to call our 'packaged in-service course.' Our theory then was that we could put together written material and film so that teachers would watch a three-minute section of film, then try teaching that lesson themselves, then possibly watch the film again (depending on what it was), and then talk about it. By devices such as taking 30 teachers in the room, and breaking them up into ten groups of three and having *a* teach the thing to *b* and *c*, and then *b* teach it to *a* and *c*, and so on, within ten minutes every teacher in the room had a chance to try teaching that particular topic. Our hope was that by this kind of device we could take anybody (like the principal, or someone like that) and have him conduct these study sessions.

We probably had about 18 different places where we tried out this 'packaged' course, and in all but about three of them, somebody there knew the Madison Project already. Indeed, that's how the try-outs were arranged, because some teacher would go back to a school somewhere and then say, "Hey, people out here are interested in what I've been telling them about the Madison Project. Could we use your In-Service Course?' In the places where there was somebody already familiar with the Project, the 'packaged' course worked all right. In one of the three places where there was not anybody who knew about it, it clearly just didn't do what they wanted it to do. I think the failure here was one of previous expectations. Somebody just wanted the course to do something which it didn't do. I think the main answer to that difficulty would be to provide a better description of what the course does. Its goal was a bit like this: if a teacher wanted to take the arithmetic program and mix in some analytic geometry and some algebra, we wanted to show them an easy way to do it. That's what the course was for. This particular place--where the course disappointed people the most--really wanted to use it to teach arithmetic in a very traditional way. Well, that wasn't what we made it for. Anyhow, we ended up by concluding that it really worked lots better if there was somebody present who knew about it.

Suppose you *do* need to have an 'expert' present. That does not bother me, because the arithmetic tends to work out that you really *can* have somebody around who knows about the Project's work. By now there are Madison Project experts in Los Angeles, San Diego, Berkeley, St. Louis, New York City, New Orleans, Connecticut, Illinois, Colorado, Pennsylvania, Great Britain, Canada, Japan, and even Hungary. Most of the time, if you need one you *can* find one.

One of the places where the 'packaged' course worked extremely well was in St. Louis. We picked up so many good teachers there that we automatically had another eight or ten people out of that batch who were really quite excellent and whom we could draw on in the future. We have, ever since.

At the end of what you might call 'Stage I' we had perhaps 50 excellent Madison Project experts. Then we started Stage II. Evelyn Carlson persuaded us to try to work with 18,000 teachers in Chicago. The arithmetic there was supposed to be that we would work directly with 600 carefully chosen superior teachers, and after they were trained they would ultimately try to reach the remaining 18,000. That never really came off, for a variety of reasons. However, we did pick up more of this kind of arithmetic of multipliers: we would bring in about 40 experienced Madison Project 'expert' teachers, and try to reach about 300 local teachers by a variety of devices, including using In-Service Course I, letting the participants teach kids, having demonstration classes, doing some closed circuit T.V. work, and a variety of different things like this for, usually, a two-week workshop. Then we would try to follow the participants during the academic year. When they were ready, we tried to arrange things so that each participant could try to teach about 30 new local teachers. This 'multiplier' effect can be pretty powerful, but it has its limitations.

Of course, from each new city we tried to recruit a few of the very best participants to go along and work in other cities in the future.

We started something else as well: we tried to staff every classroom with two teachers instead of one. This made it possible to pair up competences: a strong mathematician, say, paired with a creative primary-grade teacher. It also provided a kind of 'spare tire principle' or something like that. Since we were getting into big numbers we couldn't be sure we were right about everybody we selected as staff. But if there were two people in a room, we reserved the option of trying to tilt the balance one way or the other as time went on. You don't really say, 'Could you be the leader and could you be the assistant,' but you try to find a subtle way of saying the same thing: like, 'Could you work on the geometry and let him do the applications to the local curriculum?' You manage, in effect, to select the stronger staff member for the dominant role. That gave you much more chance of succeeding, and you usually needed all the help you could get.

Probably our biggest successes have been in California, and even though the Madison Project itself will probably terminate any direct responsibilities out there, our teacher education program is moving ahead very successfully, under the leadership of George Arbogast, Mary Dahle, and Charles Wilbur in Los Angeles, John Huffman, Jack Price, John Gessel, and Donna Doyle Johnson in San Diego, and several other people scattered around further north, particularly in the Bay Area. That's one of those sad goals you have to aim for: the program is going beautifully, but they don't need us any more to help. They have plenty of well-qualified local talent. (Of course, the California State Legislature has been very cooperative, and so has the State Department of Education in Sacramento.)

So . . . we really do have the arithmetic of dozens of people reaching hundreds who reach thousands. Since those are all *teachers*, you are potentially reaching many thousands of children. And we've been in operation so long that some of the *children* we taught are now themselves becoming teachers!

In a way it's sort of weird because our original premise was: don't rely on printed materials but rely on fact-to-face contact. You actually have students and meet with them. People originally said that's just crazy arithmetic, but in fact that's roughly the arithmetic we're still using. You just have to get a few multiples --I mean you can really deal with 50 people. Now you don't have to multiply by 50 too often before you've got a lot of people. So that really does tend to work, but it has the disadvantage that it makes what we do that much more obscure, because everybody's doing things slightly differently. There was an in-group joke out at Webster College which you could already guess: 'This sounds awfully like the original spread of Christianity or something.' And we have roughly the problem which indeed they have had with Christianity--there are lots of different flavors of it by now and you're going to get different versions. We have schisms, pseudoschisms and potential schisms. It's sort of fun.

Q: *This leads to a question I have: what do you think the major misuse of the Madison Project is? Any time you have a project like this, there's a good chance that somebody is going to misinterpret it. What are some of the things that you would suggest people guard against in using Madison Project material?*

A: I'm not sure that I've seen any misuse of the Project that I don't think would probably have happened to quite a wide range of materials. I've seen bad teachers using our methods, but I think that same situation would have been just as bad if they *hadn't* used our materials. I don't know anywhere where we probably made things worse.

I'm using my judgment when I say it was a misuse, but I think it's almost always been a situation where the teacher and I could probably never agree on what was desirable. That is, her philosophical assumptions and mine are so different that it's unlikely she'd approve of much of what I did and vice versa. In fact, some of the people from Colorado who were in on our trials when we tape-recorded lessons in order to get judgments on them, concocted the nice stunt of classifying teachers into two groups according to which lessons they would like from among the tape-recorded lessons we'd collected. This classification worked very nicely. One of the psychoanalysts working with us used to talk about two groups as the 'super-ego-dominated ego function group' and the 'id-dominated ego function group,' which is pretty exotic to some people's ears, but whatever you want to say, there are some people who really are primarily concerned that things must be much more orderly and *right*, and there are other people who are much more concerned that things need to be more creative and more open. I myself tend to call these the 'ought' people vs. the 'growth' people.

For the 'ought' people you get the feeling that an abstract ideal exists, like a mold or a Procrustean bed, and they believe the teacher's job is to make the child fit into this mold. If he doesn't, you shave here, and trim off a bit there, until he more-or-less does.

By contrast the 'growth' people think of a child growing and learning in response to inner urges, perhaps building cognitive structures in his mind by quite personal interpretations based on his unique collection of past experiences, and seeking out and fitting in new information wherever the gaps in his mental structure cry out to be filled in.

Whatever you call it, you could take a tape-recorded lesson and confidently predict that the 'ought' group would love it, whereas the 'growth' group would hate it--and you'd be right. For another lesson you could confidently predict the opposite responses, and again you'd be right.

I think probably the moral to it is that nobody should ignore these underlying philosophic assumptions. If you believe, as I tend to do tonight, that you learn the culture you're in, then you can go ahead and build educational experiences on that foundation. I think a kid who's around people who talk English will speak English. If you're around where people do a lot of mathematics, you'll learn to do mathematics. I don't feel much need to tell children to go drill or practice or something because I think they learn the things that are valued in their culture. Where they need to practice a specific skill, they'll usually do it, the way kids will go and throw a football around. The Kennedy kids did grow up to play touch football on the lawn. I don't think you have to say to them, 'Now it's time to go practice touch football on the lawn.' If it's there in the culture they'll do it. This doesn't mean we can just be passive. It means that, above all else, we must work very hard to create a viable culture in the school, and we must arrange things so that a child will feel that that culture does have a place for him, a niche that can be his.

I'm not arguing for the truth of my assumption. I'm just arguing that I think people ought to identify their assumptions and if that is your assumption, then you're probably not going to be able to pull off a stunt where you say, 'Now go practice touch football on the lawn.' Every time I say that to kids they don't do it. But there are teachers who go on the assumption that kids do exactly what they're told, and those teachers seem to be able to make their approach work.

Q: *You 'buy' the fact that there are different types of learners though --different types of kids for whom that kind of program may very well be the best one.*

A: Right. I certainly don't mean to minimize that. Really, we all know that only very selective schools are succeeding with a high percentage of their students. So if you find you're failing with some of the students, there's nothing new about that. I don't really say that so much in defense of the Project. I think there are terribly serious problems but I don't see them lying in anything specific that we're doing. The problems that bother me are that I don't understand really what makes people learn. I don't understand what makes them *not* learn. I don't understand what makes some schools not work. And there are not only great individual differences between students, there are incredibly great variations in what the same child will do on different occasions, and especially in what the same child will do in different environmental settings.

Q: *You've swung very heavily toward a math lab approach--just what's your perception of the probability of being able to get in-service teachers to use this approach? What will we see in ten or fifteen years as far as math labs in the country? Will many be here to stay?*

A: Well, certainly they're coming so fast it's almost become a fad, but on the other hand it might turn out to be a fairly permanent one. I think that wouldn't bother me but of course, as you probably know, Geoffrey Matthews is already going around England now saying that the question is, 'Is there any math in the math lab?' Actually, the first look I had at math-lab type school experiences was in the Syracuse, New York area at the seventh and eighth grade level, over ten years ago. I didn't like it. It seemed to me that the kids were just playing with all this stuff and not really learning anything. But it's interesting that some of the tasks they were doing were the identical tasks that you see nowadays in the modern films, especially from England, and that everybody nowadays tends to use--pouring sand or water from one container to another, and so on. When I first saw this ten years ago I really disliked it a great deal. The first person who tried to persuade me that geoboards were a good idea was George Polya. I thought then that he just had to be out of his mind. Obviously he was right and I was wrong. I guess that's life.

My original objection to all of this 'math lab' work really stemmed from a single source: I had in mind an abstract mathematical structure which I had taken great pains to erect, in years of study at M.I.T. 'Points' had no length, width, or other dimension, but if you deleted one point from the surface of a sphere you changed its topological properties very drastically. All of the rationals formed a set of measure zero, and so on. This was a very elegant abstract model, and its most conspicuous feature was its tightly cohesive internal consistency. For all internal transactions, this model *worked*! But what would happen to it if 'points' were replaced by brass *nails*, and if line segments were *rubber bands*? The model would be totally destroyed. All the nice theorems about limit points, open and closed sets, and so on would be utterly destroyed! Who needed that?

Of course I was wrong. I was comparing physical objects in the real world against an abstract model in my mind, and the two refused to fit together. But geoboards weren't there for *my* benefit. Ten-year-old children have not ordinarily studied topological spaces, they haven't built this vast elegant tightly-cohesive abstract mental model. We can't destroy it for them. Quite the contrary--experiences with physical reality early in their studies can help them to *build* this elaborate abstract mental model. It still seems like a miracle that such a process should succeed, but the fact remains that it does.

There was a discrepancy that bothered me as a child a great deal--that the teacher would frequently ask me to do something, but when I complied she would reject my answer. At first I thought math labs posed this same difficulty for children. It seemed to me that if I were one of these kids, I wouldn't believe what was happening. If I pour pints into quarts it never comes out even. If we want to know whether these glasses all hold the same, they won't if *I'm* pouring it. So instead of ever confirming any of these simple 'theoretical' ideas, I always proved that in fact the real world didn't work that way. In fact, the labs that I had at M.I.T. in chemistry and physics always proved (to me) that the lectures didn't describe physical reality at all. I remember sliding blocks down inclined planes; it never did work out the way the book said it ought to.

Well, that put me off all lab-type work--really quite unnecessarily so. Later on I got a chance to work in better settings at I.B.M. and at E.D.C., and that's probably what I'm trying to do with the Ithaca teachers.

When I went to work for I.B.M. for the first time I enjoyed working with apparatus. There was something about their attitude towards their computer (it was the 704 in the days before Fortran) that made everything all right--very enjoyable, in fact. The I.B.M. people seemed to radiate a belief that it was a job, and they expected things would work. It's maybe a little like the attitude you feel pilots have toward aircraft or something. They felt the machine would probably work and they weren't really greatly worried about it. This was terribly different from the high school and college labs I'd been in where you had a half hour period or a two-hour period to do an experiment and you were supposed to get 'good data.' You were supposed to get good data, go home and work it up, and come back and have people look it over.

Robert Rosenthal has written some very good comments on this in his book on experimenter effects. He suggests really people doing experiments should not know the outcomes because the whole point of an experiment is to ask a bona fide question of nature. *Whatever answer you get, if you can substantiate it, should be acceptable*! The plain fact is that the forces don't work according to Hook's Law and the like, because springs are not that perfect, and inclined planes have dust and grease and all kinds of things on them, and if you pour water you get surface tension effects. These things just don't work that well. But whereas early lab experiences drove me away, my later experience at I.B.M. left me very happy about the mis-matches between physical reality and abstract models. The discrepancies can be fun once you stop being afraid of them.

Later on I taught a course jointly with Bill Walton at Webster College, in which we combined math and physics into a single course. We let the kids really take their time on everything. One of Bill Walton's remarks was: 'Any experiment worth doing is worth doing many times.' I must say I really believe this, because you do it a few times to find out how not to do it, to improve your apparatus, to perfect your technique. You know, suppose we're pouring water. You find out we're losing water because the inside of this glass is still wet. So we didn't pour it all. We've got to figure out if we can do anything so that all the water will pour, or, alternatively, can we find a way to estimate how much water is left wetting the inside of this container. So now we've got a new experiment. You have to keep fixing it up. Sooner or later you've got an experiment that actually gives you some numbers you've got some confidence in. It's magnificent!

But this is all so different from what happens in the typical university lab, where you work under great pressure of time. In our course at Webster, we had the lab open so the girls could come in and work as long as they wanted. There was no pressure of time. They could do these experiments over and over again. If they thought the best stunt was to do it a lot of times and average the values, they could do that. They could do all kinds of things like this. For me that was a very pleasant experience; I don't know what the undergraduates thought of this--they didn't like it as much as Bill Walton and I did, I'm afraid. The thing was it gave experiments an integrity that they'd never had for me before. Previously, experiments had always been sort of a gimmick. Now I came to believe that if you really worked at it you could actually ask a question of nature and get a reasonable answer. It won't be perfect, but it'll be reasonable, and you can keep improving on it if you want.

I had some other experiences like that. At one of the Cambridge Conferences, we had some scientists there one summer. I was working with some very good biologists and physicists, and again it was really a great deal of fun. But it was very different because they didn't know the answers to lots of things and they knew it. For example, there are on the order of a few billion different kinds of molds. People have given up any attempt to classify molds, I understand, because there are just orders of magnitude too many. And so molds would occasionally get in some of the biological stuff and contaminate it. The biologists made no pretense they knew what the mold was. In contrast, in the high school and undergraduate settings, usually the teachers sort of pretended they knew the answer. At the Cambridge Conference they didn't. It's a very different attitude.

Q: *I think it was Begle who said though that one of the things that concerned him about math labs was the apparent lack of organization or structure that they give to mathematics.*

A: It's hard to respond to that. Hammersley is by implication fairly critical, among other things, of S.M.S.G., very critical of America. One of the things he criticizes is that we've gone nuts on the subject of the structure of mathematics and he says he's not aware of this view ever having been all that useful. It bothers me depending on what you mean by 'the structure.' I think the thing that Professor Riedesel did with *tens* today was quite a nice stunt. He really allowed the abstract manipulation of symbols to carry him into new areas. People have always done that. Didn't Maxwell originally get some partial differential equations that convinced him of some things about electricity before he detected the phenomena? And I know displacement current was discovered theoretically. I guess you shouldn't say discovered: it was postulated theoretically because you needed it. Similarly for neutrinos. Then people had to look around a long time to see if they could find such a thing. So I don't say that you don't allow theoretical speculations to carry you somewhere. I really do think that the ultimate formulations--the ultimate 1968, or '69, or '65, or whichever year it might have been --formulation of things, has sometimes been given too much emphasis. People talk too much about ordered fields. Some of the 'structures' we get excited about today will certainly seem rather dull and uninteresting a few decades from now.

Q: *Perhaps sequencing is the word rather than structure. Isn't it important to teach in a systematic fashion certain parts of math?*

(Interview with R. Davis -- C. A. Riedesel and M. N. Suydam)

A: Obviously I think sequencing could be important but presumably any good teacher is going to sequence. My basic assumption really is that human intelligence applied to reasonable problems can produce a reasonable response. If you put reasonably intelligent teachers in a situation where there's a lack of structure, sooner or later they're going to realize this is not working because we haven't got the right kind of sequence here. That's like the sign that says: 'Next week we've got to get organized.' They're going to say, 'Gee, we mustn't give them this before they do something else,' and so they'll change the sequencing. If you say that's sacrificing a generation or so of kids, I really ask you to look at what schools do right now. I think they sacrifice vast numbers of kids. I don't believe anything any of us will propose will increase the number. Obviously I agree that math labs can be misused. I've watched people do a lot of things that depend on the concept of function, although they haven't established the concept of function. They start doing stuff in the lab and it doesn't seem to have any payoff and they can't go anywhere. But if they've got any sense and are in situations where there may be some people around to help them a little . . . .

Q: *They need the background.*

A: Yes, certainly! Some people criticize what's happening in England now. They argue that British teachers are exceptionally responsive to suggestions. If they're asked to do math labs, they really go back to their schools and try to do it. But--according to these critics--a lot of the British teachers still don't know why they're doing it. Suppose, though, that somebody comes around to help, perhaps by asking 'Why are you doing that? What do you want the children to learn from this?' Now you're going to get somewhere. You still have problems, but I'd say hopefully a more *promising* set of problems. From a math lab I think you could get somewhere. I certainly don't mean to say that I think the math lab is the answer to all of our troubles, but it might pose the tasks for the teacher in a more interesting way.

Q: *What about the role of the computer?*

A: As you would guess, I think computers are a great thing.

Q: *One of your students cited a statement from you that in five years you think that many schools will have a teletype terminal, at least.*

A: I didn't make that up. That's actually from the management of one of the computer companies. They claim they're arranging their financing on the assumption that five to ten years are the limits.

That is, that nobody's going to be into schools in a big way in less than five years, but that some people will be into schools in a big way within ten years. If I.B.M. or somebody really got into schools heavily in, say, three years, that would be a pretty big beat on the competition. But I'd say if they're not really selling computers to schools one way or another within ten years, some of these outfits are going to find their financing is not going wo work anymore because they have been raising money against that expectation. If anything, they're going to have to worry more about the five-year limit because, for instance, I think I've heard that in something like 25 percent of the high schools in California the kids have some contact with computers, and in Southern California it's a lot heavier than that.

Q: *How do you see their usefulness? What's their potential?*

A: I really come back to this applying human intelligence to reasonable tasks because that was a big hang-up for me when I was in school. I didn't really learn things like geology because I'd get worried about who was there when they were digging the Grand Canyon and how does anyone know what would happen in that many millions of years? I'll believe people might know what happens over a few hundred years but can you extrapolate from a few hundred to quite a few thousand? So I just didn't believe it. This 'disbelief' obstacle bothered me even in physics and chemistry to some extent. You can talk about this table being made up of molecules, but I wanted to know who has seen those molecules? At that point I never got satisfactory answers. Later on when I went to M.I.T. I didn, but I really did not when I was in high school. I'm not disposed to agree that something is made up out of molecules just because the teacher said so.

Punctuation bothered me considerably, as it still does nowadays, because it's irrational. Consider the sentence:

> I certainly don't feel that I can just come out and
> say 'How old are you, anyhow?'

The punctuation there is logical, because the question is enclosed within the quotes. But, by way of contrast, consider this:

> Why can't we just tell them 'You've got to stop that
> immediately!'?

I've punctuated it the way logic would seem to indicate; but traditional practice would have it otherwise.

In high school I kept finding things that were either hard to believe or quite obviously wrong. If you stay in school, you learn to play the game and you do learn to put the question mark where people tell you to. My children use the common-sense past tense--

my seven-year-old daughter will say, 'I goed out' or something like that. I don't feel too disposed to correct her because I really do believe in Shaw's argument that the English language is simply wrong. The past tense of 'go' is clearly 'goed.' Anybody can figure that out. Any seven-year-old can. Sooner or later she's going to find out, 'Well, that's what it *should* be but the grown-ups are always doing these things wrong and so . . . .'

Now this is a problem computers can help to solve. They can provide a context within which you *do* mathematics as a creative thing. Somehow, in a situation like that all these problems of 'credibility' or 'things not coming out right' become trivial, unimportant, and much easier to live with. Making the computer do what you want is a sufficiently engrossing task that you can live with minor doubts and minor frustrations. In a really pedantic atmosphere you can't. The computer is a consistent machine which, in its own terms makes sense.

One of my friends is a linguist and looks at it linguistically. I don't know enough about linguistics to do that, but he would say that the computer language possesses linguistic consistency in ways that the English language doesn't. If one more certain kind of mark on here should do something, it will. There's not going to be an arbitrary convention saying that it won't. At least, usually there won't be . . . .

Q: *Does this mean then that you would see youngsters writing their own programs and trying this thing as opposed to a tutorial type of CAI or drill and practice type of CAI?*

A: Emphatically yes! Indeed, the Bolt, Baranek and Newman effort is aimed at having the kids write programs, even as early as the second grade. It works! They have games the kids can play with the computer in order to get used to the keyboard. The point ultimately, which I like, is for the child to be in charge of the computer. I like the 'personality' of what they're doing and I just think it's sort of a 'nice chemistry set in your basement' kind of thing. That's what it should be.

Q: *At what grade level would you start this type of interaction between the computer and the child?*

A: Well, I can tell you what Bolt, Baranek and Newman has done. Their first experiment was in Concord, Massachusetts, in grade twelve last year. Then this year they've been working in grade seven in Lexington, and now they've just started a few months ago in grades two, three, and four in Newton. But the minute I started showing it around Cornell a lot of the people here wanted it for pre-school kids. Well obviously it's a symbol gadget--you can use the symbols any way you want. They can be as simple symbols as you want.

Q: *Have you found that the computer is not always as logical as it's supposed to be?*

A: Yes. I think that's quite right, though I do feel differently about the issue of irregularities in nature than I do about irregularities in human conventions. There's no reason why people can't make an agreement and keep it within reason, but certainly particularly here where we're using telephone tie-lines there's a lot that can go wrong. We're using acoustic couplers. The computer will misbehave occasionally, but it seems to me that's one of the things you do need to learn, that the computer is logical but it makes mistakes.

You asked me a question before about what should the Project do next--I didn't answer that really for the Project. Imagining a lot of funds available, what I think should happen for education in the United States (not by any means just the Project), would be these very different experimental schools. There could be things less extreme than that that I think should be available.

I'll go to the opposite end and give you a modest answer to that. Our budget gets tighter and tighter for a variety of reasons, but more and more people are interested in what we do. You probably know about this conference that was held at the Smithsonian Institute's Belmont Conference Center in Baltimore recently. Several participants talked in terms of an agricultural model that might have implications for education. They said in effect 'Perhaps what you need is some kind of county agent.' Something like that honestly makes sense because all Projects get many requests that we could respond to and we would like to and we should. They range from individual college students taking a methods course somewhere to people calling from the State Department of Education in some state asking us can we come in and set up a state-wide program. We could do a great deal with that but we cannot do it unless somebody pays for it. Right now I don't know how to get that financed. We're really trying several attacks simultaneously.

The modest answer is, let's take some of this expertise which we really have acquired. While I make fairly restricted claims for how much we know, there are things we can do and we know teachers who can do them successfully and much more could be made of these people. We seem to be, if we're not careful, about to lose them just at the point where they are identified. That would be a shame. We have a definite group of experienced, identified teachers. One could build on that. That's the modest thing.

There are some intermediate things that I'd like to see the Project get into that fall between these two extremes. One, certainly, would be undergraduate education of teachers. As you saw with the course up at Syracuse, I really am working on that anyhow --on Syracuse's money--but I'd like to see if we couldn't do more of this sort of thing. I think the tack I'd like to take on undergraduate education would be to put together three things: learning mathematical content, dealing with kids and learning to make effective use of physical materials. Hopefully one could combine all three together, and not feel so tied to an *a priori* curriculum that one had to ignore the actual students.

There is great potential power in any situation where one is able to take cases on their merit and not feel compelled to say 'this is the English class' or 'this is supposed to be' . . . whatever it is. We nearly always have a frame of reference and all business has to be transacted and interpreted within that frame of reference. I was recently at a meeting arranged by the U. S. Office of Education, where they had some high school kids present giving their views on what's wrong with high school. None of them complained that their courses were not good enough in the sense of content, which surprised me at first because really we know they're not, in all too many cases. But then you think about it: how could the kids know about weaknesses in the content of their courses; that's the only content they do know. But many of the students did complain that 'the teacher won't discuss what we want to. I mean, today the astronauts are in orbit and we want to talk about the astronauts. But if the teacher doesn't see how that fits in with Shakespeare or something, we can't talk about astronauts.' I think the students have a valid complaint for several reasons. I really don't see that we need to be that constrained to say well, after all, everybody's got to know Shakespeare. That just isn't all that clear to me. Actually, if we make a list of the things everybody's got to know, the first thing you find out is that most people don't know it anyhow which cases doubt on it. So that really would be one of my directions to go to--undergraduate teacher education. With ten years of experience working with elementary school children and with in-service teacher education, I think we are now in a position to create a vastly better program for pre-service teacher education at the college undergraduate level.

I think you had a good phrase for it--I'd like to get them so that they're sort of motivated learners for the future. With any reasonable success, we ought to be able to provide future opportunities for them because there are in-service courses, there are state department things, there are correspondence courses, there are summer programs, there are N.S.F. institutes, there are Saturday, after school, evening programs, there are all kinds of places you can go if you want to study. So if I turn out a teacher who doesn't know something, that doesn't bother me, if they're going to have a potential to grow. That seems to me to pose a very interesting problem. I hope measurement experts are going to think about the possibility of studying not what someone knows right now, but rather what happens to him over the next ten years. Does he grow? And if he grows, then he must have had a reasonably good beginning. If he stagnates, then something went wrong.

Q: *Except you have to have some base for finding out whether what you did was right or not, to be able to replicate it if it was right. You want to reproduce the same set of circumstances for somebody else and for this you need more controls.*

A: Who can argue with the desirability of that? But the possibility of it is something else again. My guess is that the variations in any one individual's mood, added up when people are combined in a group setting, and with further allowance for variations between different people, and myriad parameters describing the setting . . . how could you ever describe it? How could you ever hope to duplicate it? Can any of us ever again feel the way we did before John Kennedy was killed? 'Repetition of conditions' is a sterilized laboratory abstraction that probably never applies to real situations with real human beings. Are the nuances between black people and white people the same as they were four years ago? Yet all of these things are very relevant to education. Could two schools ever be the same? I guess I would say two mediocre schools might be the same, but a really good school might be impossible to duplicate at any other time or place.

Q: *You want to be able to say, 'This is exportable, we can do it this way now and get those kind of results with this way.' You have to know what your variables are or have some image of what your pertinent variables are in order to continue it.*

A: Of course I again agree with the desirability of that, but on the other hand is it not also true that you go a long time before you usually do know what you are actually doing. One consequence of my living in St. Louis is that I got to know some brewers, and they interested me in the history of brewing. You know lots of people made lots of beer for many centuries without really knowing what processes were involved. Do we actually know very much about learning? It seems to me that anyone who wants to answer 'Yes, we do,' is then compelled to explain why, that being so, we can't eliminate illiteracy, why we still have divorces and suicides, why organizations so often disintegrate under the conflicts of interpersonal relations, and why we can't all write music like Prokofieff and do mathematics like Paul Erdos. Of course we can teach mediocre skills under favorable conditions--but does *that* represent the true power of our 'understanding?'

Q: *Maybe you're really talking about the first stage. At the first stage you do just have to see what seems to be working; and then you hit a point where you can say maybe it's this, maybe it's this, maybe it's this. Now let's go into it with firm controls--experimentally.*

A: Well, I'm not all that sure that 'experimental replication' will ever apply to human behavior beyond the most trivial level. I don't believe I hate any attempt to deal with reality in a rational way--I have spent ten years studying behavioral science when conceivably I should have been doing something else. I don't begin with an *a priori* hatred of behavioral science. But it seems to me honestly that behavioral scientists are being--the only word I know is arrogant--in trying to move so fast. I would claim Shakespeare probably had a deeper understanding of human behavior than most psychologists do, yet they would claim they're going to make progress ahead of him. I don't find them even catching up with him. They might say our first task would be to understand things as well as Shakespeare did and having done that we'll try to go on from there.

Really this is not inimical to what any of us might do to understand education, because we're not compelled to some particular conceptualization of the task. If you were going to study sculpture, how would you study sculpture? You'd surely use different media. You'd almost certainly have at least photographs of it. Now a two-dimensional photograph is usually a poor reproduction of three-dimensional sculpture but you'd probably do at least that. You wouldn't settle for a verbal description, or for three numerical parameters.

Q: *Most of your comments have been toward very much of an unstructured curriculum. Don't you think there are a certain number of basic skills that we should in some way 'motivate?' I mean, maybe set up situations where we 'hook' them into wanting to do it? By the time they finish elementary school if they're normal children, don't you think there are certain basic skills in terms of reading, language, and mathematics that the kids should have had a pretty good opportunity to master so that they can go to the next step themselves?*

(Interview with R. Davis -- C. A. Riedesel and M. N. Suydam)

A: I seem to be much more reluctant than I mean to be but I think again that's an awfully deep issue. One of the principals down in Philadelphia where I just visited was raising this question. She said, 'We've got fifth and sixth grade kids who just can't read at all.' She didn't feel that she could really go back and teach them the basics of reading because there wasn't time. She had to keep aiming for something like a sixth grade level. Now maybe not really a sixth grade level but she felt under some compulsion to do this. She herself said, 'I think I might be better off turning out a child who graduates from the sixth grade reading proficiently at the second grade level and admit it.' But she said, 'I don't really feel free to go back and do this.' That's the reason that I show some reluctance here. In intelligent hands you can say almost anything you want. If it's handled intelligently it's going to work. The problem is we know it isn't always handled intelligently. What's going to happen as we get more 'accountability of' schools? You can see people saying, 'Why that school is graduating kids who read at the second grade level.' Now there ought to be a response to that, yes, but they *honestly* read at the second grade level! They enjoy it, they'll do it! Given a book at that level they'll pick it up and read it voluntarily! They've really got that! I certainly think this is preferable to these kids who sort of make a pretense of reading at the sixth grade level but really everybody knows they're not making it. That's the reason for my hesitation. What impressed me very much was teaching freshmen at M.I.T.; theoretically they had to know trigonometry, for example, before they entered M.I.T. Some kids came in not having trig. It wasn't necessarily a disadvantage. You got to some formula they didn't know. We had a standard routine for that; we had this little black book called *Burrington's Tables*, which had all of the necessary formulas in it. We'd just say, 'Okay, why don't you take *Burrington's Tables*, it's got about three pages of trigonometry, go study it over the weekend' and that would usually do it.

Q: *Of course, those students were about two standard deviations above the mean.*

A: Right. I agree to that, but on the other hand it tended to give me the feeling that I have about what people can do *when they really want to.* Not all university professors would agree to my view--some would say, 'Well now, we can't teach anything to a kid who doesn't know thus and such.' Personally, I have yet to find that essential 'thus and such,' the real *sine que non.*

Q: *In math you find that's particularly true.*

A: I certainly think so. The people who are so sure of the one true sequence usually haven't practiced much creative search for alternatives.

Q: *Well, just like, for example, there is a course we have called Geometry for Teachers. It's a 400 level course and you have to have the five-course undergraduate calculus sequence to take that particular course although there isn't any relationship between those two courses to speak of.*

A: I would certainly think that's an example. Another example would be where geometry should come in the high school program. In *most* American schools it comes in the tenth grade, but in the private schools around Boston there's a tendency to put together the two years of algebra in grades nine and ten and let geometry come in the eleventh grade. That's a different pattern but as far as I know there's no real advantage to either sequence. Not enough for either group to convert the other. Most European countries do not teach Euclidian geometry at all and don't find that disadvantageous. It's awfully hard to find that central stuff that 'everybody's got to know.' I presume you keep hearing about this alleged central minimal knowledge, because I do too, at faculty meetings and elsewhere. Teachers say, 'I can't possibly teach calculus to somebody who doesn't know analytic geometry.' Well, M.I. T. never used analytic geometry as a prerequisite for calculus in all the time I was connected with their program. So apparently you *can* teach calculus to people who have no previous knowledge of analytic geometry.

Q: *But on a very early elementary level, you will agree there are certain things there that we do need as a basis. For instance, the whole matter of that child who didn't recognize the numbers or couldn't count: here's something we do need to teach as a base.*

A: I agree with that. But you see my main point. I just worry that we're going to get rigid specifications a la the Department of Defense. I'll tell you what I particularly worry about--suppose that someday there are private corporations or various outside groups running schools. Now basically, I like this idea. I really do like the matter of choice. I can choose to buy a Volkswagon, and if I decide it blows around too much in the wind, I can decide next time it's going to be an Oldsmobile. I have that choice and I like it. The only place I know of in our life where we don't have a choice is in schools.

Q: *Yes, that's true.*

A: Now I find that very unacceptable because I think schools are *more* important than automobiles. If they told me I've got to drive a Chevrolet, I could reconcile myself to that more easily than to saying that my kid has got to go to this kind of school because it's the only kind there is around Syracuse.

Q: *Then the school's competitive, so to speak. 'We want to do better than these.' Well, of course, it would improve. We get this from some of the suburban schools where people will move in and out of different communities based on the school system.*

A: Right. You understand exactly what I mean. I think competition could be tremendously beneficial. But now, suppose it happens. There's going to be much more of this issue of accountability to determine who gets funded or who gets discontinued, or when have people completed their contracts. If this gets to look like the Department of Defense approach to accountability, you're going to get people trying to make the specifications in such a way that an accountant or lawyer can decide whether the contract has been fulfilled. Now that's another kettle of fish from your being able to distinguish that you don't have to know trig to do calculus but you do have to be able to count to get very far in arithmetic. We can put priorities on these things and I think we can agree on them. But if I were talking to a lawyer or an accountant about this, I think you're going to have a great deal of trouble specifying in these documents what's going to be required. *What bothers me is that the specification isn't being made for educational purposes, it's being made for commercial purposes.* You know some of the studies of the Department of Defense don't make them look very good either. They are not a triumph of effective use of the taxpayers' money.

Q: *A lot of the conversations about manufacturers moving into this area --of systems of some kind being sold to schools--are based on the idea that they will have behavioral objectives, behavioral objectives by definition which can be measured. So wouldn't this clear up your whole question?*

A: No. I think that's going to make exactly the problem. I think that's exactly the wrong way to do it. Those behavioral objectives are being put in for commercial reasons, not educational reasons. Zacharias once said, if you'd allow infinite lists of behavioral objectives, maybe you could do it. I talked once to some devoted 'specify your behavioral objectives' folks. One of the questions that came up was this: I said, 'Now I visited Westminster Abbey. I found this a very moving experience. I've been there several times. I visited Coventry Cathedral. I found this a *very* moving experience, but I can't describe in words what I got out of it. And in that sense I don't have any particular objectives for it. Would you put this experience into your curriculum?' And their attitude generally was, 'No, it's too vague, we wouldn't.' Now I totally disagree with them. Those visits were experiences from which I learned a lot. *I* know I did. If observers of my external behavior can't see any difference, then they are unable to describe my education.

Q: *Popham was talking at A.E.R.A., he's apparently changed his view on the behavioral objectives. He has another set of objectives that he calls experience objectives, which specify a youngster will have an opportunity to do this, this, and this without any behavior at the end. In other words, a lot of the math lab objectives are experience objectives. The child should have this many experiences with the geoboard; now one of them learns area, somebody else learns the triangular shape, and somebody else just played with it, but maybe next time they'll get some insight.*

A: Now that begins to make honest sense. I think that makes honest sense because there are things one should have done. Suppose I had gone to Westminster Abbey because somebody told me to, because I had a list of things to check off like find the names of seven important people who are buried here. I bet it would not have been as moving an experience as it was simply to be walking around London and think, 'Gee! There it is! I could go in!'--and I did go in. Really there are some experiences you ought to have, even if fallible humans can't describe, in 1969, why you ought to have them.

Q: *Very often the most important ones can't be stated behaviorally.*

A: Yes, honestly the most important ones. I'm not kidding. I'm not attempting to be frivolous. Listening to Beethoven's last five quartets I find a very moving experience. But if I were listening to them primarily to learn the themes, if I were listening to them primarily to be able to recognize them, I think it wouldn't be such a moving experience. I would become a more shallow intellectual experience.

Q: *One of the things I always remember: I had a course with Ernest Horn who was one of the last great generalists in elementary education. He knew quite a bit about all there is in the curriculum. One of his constant comments was that you shouldn't mix skill reading with literature because you destroy literature. What he wants them to see in terms of a very good story that's appropriate for a fifth grade child, is quite different than the type of thing he wants to analyze in terms of paragraph structure or in terms of word* *structure. You destroy the literature effect by teaching skill reading. And this is the same type of thing we get in English in the colleges. You destroy a moving poem by analyzing the meter, and analyzing each line. You should probably use 'junk' to analyze that as opposed to destroying literature.*

A: I quite believe that. In fact, I think it's very important. And it seems to me honestly, if people like us do not object now, it's going to be too easy for corporations, accountants, and lawyers to agree on non-educational specifications for contractual purposes, because it's the easiest thing for them to do. The studies of the Department of Defense are *not* encouraging. They've had, in effect, behavioral objectives. The companies have in effect achieved them. But the weapons systems they prepare don't really do what we want them to do. One of the difficulties is that there's always something you've left off the list. It turns out, maybe, that if you leave the thing out for three days it rusts, but nobody thought to say on the list of specifications that it must not rust if you leave it out for three days. Again you should realize I don't mean it frivolously: I think this is even truer with human beings. The things that come to my mind always sound so ridiculous. Obviously you don't want a high suicide rate, but that sort of thing is ridiculous. You wouldn't think of putting that on the list. We don't want a teaching program that produces neurotics.

Q: *No, from an analysis of the number of kids getting ulcers in some of the heavy pressure schools, that was never on the list for improving the content of the program.*

A: Now that's exactly what I mean. I don't mean to restrict it simply to pathology but that is an example. You don't want kids who are driven to taking drugs. You don't want kids who, having learned how to read, no longer do it. One of Zacharias' arguments is 'I want to know when this kid is 35 years old and not a kid anymore, how many books does he read a year, what kind of books.' And those things get left off the list.

Q: *You named one this morning--the goal of the school is to produce responsible citizens. But this has never been an acceptable behavioral objective to most of the behavioral objectivists.*

A: That's exactly right.

Q: *This is one of the things they talked most about in terms of scientists and such--how responsible are they and this kind of thing.*

A: Right. You know very well what I mean. Although I myself am not Catholic, I have been very much impressed with Catholic schools, because they really take seriously things that we don't in most other schools. A child's self-concept, his relation to other people, even issues like sex. 'Okay,' they say, 'this is important': 'Who you are as a person, how you're going to relate to your husband and your children is important to us and we will talk about it.' Now the public schools don't deal with this. Very possibly under a state-operated school system they can't. But if you leave all these things off your objectives--well, nobody honestly wants to produce somebody who can do mathematics, but he can murder six million Jews; he can't get along with his kids, and so on and so forth--you wouldn't call that a good school, I wouldn't call that a good school. That's got to be classified as an educational disaster. I don't really see anybody coming along to Westinghouse or whoever else is running a school, and telling them 'Look, you're not making it because your graduates have too high a divorce rate.'

Q: *Now see if you'd agree with me on this. I think one of the contexts through which the teacher can many times best convey these types of things is through the way she handles the subject matter. Obviously, I think one of your goals of the Madison Project is to do this. I think devoid of any kind of subject matter you can't teach democracy as a unit on democracy. It's the way the teacher handles the classroom and the way the students do it. In other words, maybe the only reason for working in committees is not to learn the subject matter better, it's to work in committees, but you have to work in committees on some kind of subject matter.*

A: Okay, now that's one of these things where I wish I'd said that! I very much think that you're not going to teach maturity and so on by courses on democracy or by courses on how to work in committees. When you try that approach the result is invariably superficial. The National Training Labs, I think, have worked out things where they give people actual problems to solve and have them work on it. Because you can't just say 'Now go and work in a committee.' You've got to have something to work on.

Q: *We were both impressed, in terms of classroom climate, with what you can do toward the pupil-centered or pupil-acceptant type of thing as opposed to the teacher dominated. We know a person on campus who's very much that way but he deals in generalities and essentially not any subject matter. You know you can teach people to handle climate. I contend you almost have to do it in a context such as something like the Madison Project where they are thinking of two things: now how do I accomplish some of these goals as far as mathematics are concerned, but how do I do it in a way where I'm* *reinforcing the youngsters and I'm really concerned about how they feel about mathematics. You know I don't think you can do that in a vacuum without the subject matter.*

A: In fact, I'd buy that as the best statement of what we're trying to do that anybody has ever made! Honestly, that is precisely what we're trying to do. I didn't at any stage say it that well, but that really is it. And that's why I say, I wouldn't be so excited about mathematics education if it were just mathematics. I don't think it is. I think the way you learn to get along with people is by having a job to do. I was impressed with *The Russians Are Coming*. I don't know whether you saw that film. Now how did they get out of this mess when the Russians and the Americans were about going to kill one another? They got everybody working on a common job. It wasn't manipulated, but all of a sudden the common job presented itself and everybody worked side by side on this job. Suddenly the whole atmosphere changed, and nobody wanted to kill anybody. I think you couldn't have produced this result by somebody standing up and saying, 'Now let's all work together on like, whatever, well somebody suggest something . . . .'

Q: *You find the same thing happening in New York City for instance during that electricity blackout, or in a snowstorm.*

A: Right. Then New York City is a nice place to be--the nicest feeling there is always manifested during their crises. The New Yorkers actually cooperate. The rest of the time they can't get along with one another.

Q: *Well, this is true almost every place. If you put people under a crisis that they thought was a real crisis, but yet didn't produce so much anxiety that you would have bad behavior, people just become friendlier don't they?*

A: Yes. I quite agree with that. And I hope that even people who love mathematics very much will be able to see that education, in the sense we have been speaking about it, is a still more important thing for the future of mankind.

Project Title: Comprehensive School Mathematics Program
Location: Carbondale, Illinois
Director: Burt A. Kaufman

Background

The goal of CSMP is to provide a feasible solution to the teacher training problem and the classroom organization problem by totally individualizing the mathematics curriculum. Besides the development of curriculum, it includes the development of new teaching methods, a research program for evaluating the materials, and a systems approach so that the materials developed will be used to their optimum potential.

The philosophy of the project is enunciated by the mathematicians and mathematics educators who make the basic decisions regarding content. The learner is to be involved in working with a wide variety of materials and activities in a mathematics laboratory situation. The preparation of activity packages so that a learner can proceed through the curriculum at his own rate is the heart of the project at this stage. Ideas from many other curriculum development projects, as well as original concepts, are being incorporated. The intent, however, is not to up-date the curriculum, but to reconstruct it.

Each set of materials is tried out with several groups. The initial use is with only a few bright children--if the idea isn't appropriate, it is discussed and either modified or dropped. Then groups at other ability levels test the materials, with continuous revision built into the process.

A new role for the teacher is implicit: the children do a great deal on their own; machines will do what the teacher can't do as well. The teacher becomes the organizer and the tutor for the individual child. CSMP is based on the premise that education has to change, and the curriculum must be changed before the school changes.

INTERVIEW with BURT KAUFMAN, Director

COMPREHENSIVE SCHOOL MATHEMATICS PROGRAM (CSMP/CEMREL)

C. Alan Riedesel and Marilyn N. Suydam
(21 April 1969)

Q: *How did you get started on this project?*

A: Well, we were at Nova in 1963. The thing that attracted me there was the original plan, or at least freedom for curriculum development that they purported in the advertisement for the school. They were going to try to develop the school from an academic base by bringing in subject matter people, or people with more subject matter training than is usual in the high school.

We got down there and had this freedom; so I thought I'd take a few classes of exceptionally bright kids and see what can be done with them with some of the newer materials that were purportedly written for bright kids. There were some things I wanted to try out. I had just finished two academic year institutes myself so I thought a fair amount of what was in those programs seemed suitable for kids of this age. I found out rather fast that the materials that were purported for bright kids just didn't turn out to be all that good. I thought they could do a lot more and the material could be a lot cleaner. I knew the authors of the series of the materials we were using, and so I suggested that they let me try to make a revision for this caliber of student. Maybe eventually the publisher would be interested in two versions--one for really bright kids, the original for the more average kids. They said all right--they didn't know what would come of it but they said it would be fine with them if I wanted to do this.

So, I started writing, but it quickly became apparent that you just couldn't save this kind of material--the foundations were just not strong enough. Some other people on the staff were interested and we started to develop a so-called 'top track program.' We decided we would write our own materials where it seemed necessary and use any existing material where appropriate. That usually meant college-level texts because we couldn't find too much at the high school level for these kinds of students beyond ninth and tenth grade.

I had an advisory board of mathematicians and math educators which had its first meeting about one year after we began. They seemed to be impressed by what could be done under the rather trying circumstances of developing materials when you have all the normal things to do in a school system. We all became used to doing our writing late at night. They made some recommendations that we look for funds to have some planning conferences, to see what kind of implications our work so far might have. I don't think they really envisioned at that time a project as comprehensive and vast as this one.

During our first year at Nova, the Cambridge Conference published its report. That report was a true catalyst for me, because the basic pedagogical philosophy that was expressed in there was quite in line with my own thinking. This doesn't mean that I totally agree with the topic-by-topic outlines. What I really liked was the first 40 or 50 pages--the discussions. The topics themselves seemed all right at the time, but since then we've seen you can go much further if you have the right setup. I wrote to all the members of the Cambridge Conference suggesting that they might like to use our school as some sort of pilot study, to start developing and trying out a curriculum based on the report. I got four or five responses. Bob Davis visited us and was extremely impressed: he really was the person who influenced me to go further. In fact, he was probably our most ardent supporter in the beginning when we needed to have somebody with a name who was well known. Lauren Woodby, who was then at the Office of Education, visited us and gave me the specific advice of getting a small planning grant proposal in the spring of 1965; we had three conferences in the fall of 1965. Each conference lasted one week. The first two were basically devoted to content considerations, and the 3rd to evaluation and methodological questions. We had 55 people during the three weeks and the underlying feeling among all of them was that we should think of expanding into something larger, dealing with more than just the upper track student and including the elementary school.

Q: *At first you worked only with the secondary school?*

A: From 1963-65 at Nova we just had secondary school students in the fall of 1965 when these conferences were held the Nova elementary school had just opened. We were to eventually be responsible for the elementary school, but we hadn't thought too much about it. At that point we were only talking about the beginning stage of what we're now doing here--a secondary school program for bright kids. But the feeling was that the program could be extended to a larger population, and that we should take up the whole comprehensive nature of education including teacher-training, systems, evaluation, the whole works. I had never thought in such broad terms pervious to this since I was really only a classroom teacher highly interested in modern mathematics. But, I think everybody was very motivated in those days and I was thinking 'if we could have 40 or 50 thousand dollars a year for a couple of years, we might get this done.' But the conference participants were talking right off the bat in terms of three or four hundred thousand a year. As one who never had much money, I was a little carried away by all of this. Well, it turned out that's about what we're running a year, but I just hadn't thought in those days that anybody would ever trust us with that kind of money.

We wrote a report which is available, which basically gives the recommendations of those conferences. The next step was to develop a proposal. About that time all the political problems of education

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in Florida came to light. Nova University was being planned at that time--it opened a year later. They were talking about tying our project in there, but we were given a rather rigid setup about who would be the director and who would control the money: the whole thing started to look as if we could have a lot of trouble if we didn't pursue it on our own. We had, at that same time, the opportunity to transmit the proposal through the math education department at Florida State University. The people in South Florida were very adamant that it should stay in that local area and they should control the overhead. Things got very messy quickly. We finally had a confrontation with the school board and superintendent. We asked just what they wanted us to do--whether we should pursue this project at Nova or look somewhere else for a home. They hemmed and hawed and I finally decided to get out of there before it got worse. One of our consultants who was helping us to write the proposal was on the faculty at Southern Illinois University. They were rather anxious to get a project of this sort started in the College of Education here and he suggested that we come to Carbondale to present what we had in mind. After so much planning, I really wanted to get this program off the ground, so I was happy to find a university interested in housing it. We came to Carbondale for a weekend in May, 1966 and consummated the move within six days. The Nova officials and school board in Ft. Lauderdale were then upset, but they had their chance and blew it. The move was one of the wisest things I ever did as conditions at Nova in particular and South Florida in general worsened over the next few years. CSMP could never have operated in such an environment as the Nova administrators just didn't have the competence or foresight to overcome the general adverse conditions towards education in that part of the country.

So three of us came here in the summer of 1966 and worked at the University School. Our main job for the first year was to develop a proposal. We already had a beginning from what we'd done in Florida, but we got more detailed, and also started the secondary school program in the laboratory school. Six students from Nova came with us to Carbondale to complete their high school education. We took over the math program in the secondary part of the laboratory school and started doing a little something with the elementary school. It was sort of a freewheeling year.

In October of 1966 we had a conference here. We had a lot of the same people who had come to the other conferences and some additional people. We had about 30 participants for about three or four days in which we replanned the project in this new setting. The proposal was completed in December. We didn't write it for any funding agency in particular. The university was eventually to decide to which agency it would be presented. CEMREL was just starting, and I found out since that they knew all about us, but I didn't know at the time that they were going to be interested. There had been some conversation between the university people and CEMREL. CEMREL was interested in making CSMP one of its major programs which it did in March, 1967. An arrangement was made with the university so there would be some sort of joint funding for two years and then CEMREL would take over completely. This will happen July 1 of this year. In a sense they've already taken over, but there have been university contributions in terms of salaries and so on up until now. We had about $20,000 in the spring of 1967 to acquire a staff. We started in earnest in September, 1967. We started with a staff of about 15 and now have close to 30. Our projected growth is double this number within 3 years. We're now operating on about a half a million a year budget and the plans call for 3/4 of a million next year if additional funding is available. I think the projection for about three years from now is to operate this program on almost two million a year, which is what it will probably take to do the full job we envisage.

I'd say the philosophy was initially settled in those first conferences as far as the individualization aspect and the systems aspect. Since then it's been refined--the refinements have come by having brought people on staff in those areas with their own ideas. But we've always had the strong commitment to a content orientation, with mathematicians strongly involved. This is even more so now than it was before and it will continue as long as I'm running the project.

Q: *Rather concisely, what are your purposes of the project for now and the next few years?*

A: Our goals are to produce materials, and methods for using those materials. At the secondary school level our "Elements of Mathematics Program" has the objective of creating sixteen textbooks for students to use on an independent basis. These will take them through almost a full undergraduate math program by the end of high school, with a heavy emphasis on logic, foundations, and language considerations at the beginning. So one goal is to complete that program, revise it, use it, get data from it, and hopefully do some interesting research in it. We have an evaluation staff who are interested in research as well as evaluation. We have a lot of interesting questions that we'd like to have answered and so we'd like to get to some of those. With the kind of manpower we now have, and if the history of the writing pace over two years is any indication, it's going to take another four or five years to finish these books. It's just awfully hard to write this stuff. Our basic program plans show us to finish the first drafts of those books by 1973. The revisions will then probably take another couple of years.

The EM program is just one small part of CSMP. The major component is our package production which at the present time is concentrating at the grade 3-4 level. Our goal is to get at least one stream, with one parallel version for below average children through grade six by 1972 or 1973. My real goal is to get more than just these two versions. We really must get at the whole slow learner problem that exists and which we haven't attacked at all up to now. I don't think we'll have it in 1972 or 1973, but as soon as possible, I'd like a variety of approaches. There are some differences in styles of learning and teaching styles and individual differences which I think can be reflected in various content approaches. I'd like to have enough staff and time to develop more than just the little we're doing now.

Then comes the secondary program and I think that will take about ten years itself to attack.

Q: *That will be for the average secondary student?*

A: It will be for all students. The current EM program will be one track. I don't know how much will still have to be done with it to put it in a more packaged form. It might be just fine to leave it in textbook form for students who can read well and are motivated and do not need other types of materials. We might make some video tapes to go with the books, but basically it is the books themselves that carry the large burden. That might just be enough and then we're really ahead of the game because we've got one program for a certain group of students worked out. But I think there are at least three or four other alternatives and even for bright kids there are some other alternatives that ought to be developed. The secondary school curriculum is going to create many more problems due to certain restrictions. For instance, you have to prepare students in some way for examinations to enter college. At the same time there are lots of novel ideas that can be incorporated at this level. Much of our elementary school material will probably be useful for slow learners at the high school level: it's a little hard to say what's secondary and what's elementary.

Q: *You commented that to use this program teachers need more training. How do you bridge the gap between teachers not being trained to use the material you'll soon have available?*

A: Well, we're going to start training teachers this summer for the EM program. However, I never envisioned that the EM program would be used by a large number of students, at least not for a long time. We're starting out just to see what a training program should be like. We're starting with highly qualified teachers who should not have too much trouble with the content but will have to get into the spirit of the program, learn about the methodology, and also get some content training. Then we'll see how it goes in schools when we're not around. This is the first time we've tried it where we're not actually there to step in when needed. We'll get some feedback on the institute and probably expand it in future summers to maybe 30 participants. I would imagine that sooner or later we will start training teacher trainers to help with this thing, but that's a long way off.

Q: *So your emphasis is on proving the existence theorem that kids can learn this material?*

A: Yes--show that it can work in schools under certain kinds of conditions and training--then it's up to somebody else for full scale dissemination. The lab feels that the eventual dissemination and diffusion of this material is the responsibility of teacher training institutions and state departments of education. It's our job to try to get them ready to do it and to set up procedures to help them but not to actually run the whole process.

We want to keep very high quality control on the material at this time, so while we're anxious for this kind of diffusion to go on, our main goal is get feedback for revision. We hope someday to have it really user-ready in the sense that there's nothing much more to do with it. By then we hope there will be a system for training teachers.

Q: *Are you thinking that at that user-ready stage it will be a program that an individual child can take without having a teacher trained for the material?*

A: I doubt if any student can just work six years by himself--he's going to have to have help. The best student has to have help because we're pushing him to the upper limits, to a very, very high level of sophistication very quickly. Particularly the younger children who have the capabilities of doing a lot: they really do need to organize themselves efficiently. You can't just hand them the books when they start grade seven and say, here they are, you're on your own now for six years. This is nonsense. Nobody is that good.

The students we've observed really depend much less on a teacher, but when they need a teacher they need him. They need him there and if a teacher isn't available, the student has difficulty proceeding. Students help each other but they still need to have a teacher. I think they need to have the opportunity to have some interaction with teachers and other pupils. Even if they don't need the help, I think this is important. Otherwise they're going to get a certain approach to mathematics that isn't real. I mean, mathematicians talk to each other, they write to each other and so on. The ability to communicate orally is very important in learning and doing mathematics and the students should have some practice in it. Also, I think they should hear a *good* lecture from time to time as this is a way mathematics will be communicated to them in the future.

Q: *You could put together a package there?*

A: Yes. You can put a lot of things together, so that you don't need as much teacher time, but I think you're going to have to have a live person around to help them when they need it no matter how good they are. It would be very surprising if we ever got to that stage where we just didn't need a teacher altogether. I'm not even sure that that's good. I don't know how much of a teacher's time they need. I'm pretty certain that if there are one or two people like this in a school, who can do nothing else but help those that need it, then these students can work on their own and don't have to be scheduled any time.

You know a very important part of this program is grading papers and consulting with the students on the things they're doing, making contracts and so on. Assistants can be developed to help with that sort of thing. But, the assistant will have to be of high quality. You're going to need the kind of a teacher who can deal with the whole six years of this program at one time: none of us who are writing it can do that quite yet.

Q: *Let's consider elementary pupils--what's your feeling about the integration of science and mathematics?*

A: I agree with the idea, but I don't know how to go about getting science people and math people to sit down and plan a program together. The only real time I've seen it tried is in the Minnemast program, and I wonder if it's worked. I mean one time it was math-oriented and one time it was science-oriented. I think it will take a lot more understanding of the needs of each subject, realizing that mathematics is not just a tool subject or at least mathematicians will never accept it that way. They're not going to give up the integrity of the discipline, because they believe that there's a lot more to learning mathematics than just applying it to science, even though that's one of its major uses.

I am not overly optimistic about it, but it's easier at the elementary level. I think it's got possibilities there and maybe if we had our full math program and then sat down with science people to try to integrate we might be able to do so. If you try to build together, I really don't think you can--I think something gives.

Q: *Either the science or math.*

A: I think if you have a real good program it will be easier to go back and build in the applications, build in the places they need science, maybe rearrange your sequences and so on. Mathematics must keep its integrity and science its integrity. I want to see what the full limits of a math program can be. Then we might integrate as an alternative curriculum or as a blending of several alternatives, where we don't have to destroy the whole curriculum in order to get some science into it. I think you can have a pure mathematical approach with lots of applications, but I'd rather see the applications come from mathematicians who are applied mathematicians than from scientists who look at mathematics from the outside. Then I'd bring scientists in to see if we've got the kind of applications they want.

Q: *When do the applications come? After you've taught the math?*

A: I think the real significant applications of mathematics can't come until the fields that they're being applied to are also developed in the same way. Otherwise we have to teach a physics course as well as a math course at the same time. I think that there has to be a parallel type of development in the fields of application if you really want to go deeply into the applications in any particular area. Otherwise, you can deal with some elementary kinds of applications along the way. There's a lot you can do with finite mathematics in the fields of social sciences and sociology and the behavioral sciences. I think we can do interesting problems without the kids having too much background in those areas. But if you're going to do technical scientific applications in physics, chemistry, and biology, you really have to have those areas at the same kind of development level or the applications most likely will deal with trivia.

Q: *With the third graders, do you believe in starting with an application and moving to the math, or starting with math and moving to applications?*

A: At the third grade level, the whole thing is application oriented in the sense that we build most of the mathematics from situations. By applications one doesn't have to mean a scientific problem. We aren't building abstract mathematics at this level. When the children play with this cube puzzle, they're doing applications as far as I'm concerned. The whole question boils down to what one really means by application. I like to think that, at least up to the point where they begin a formal development of mathematics as an organized closed system theoretically oriented, they are involved with a lot of manipulatives--that they are discovering things from situations which you sometimes have to build artificially. There aren't enough situations in our environment to really discover all the mathematics in the world or have all the mathematical experiences we think necessary before doing formal mathematics.

There's another point, and that is that there are many students who really aren't interested in physical science applications. They're much more interested in the liberal arts approach to mathematics. I wouldn't want to ignore their interests and run the danger of turning them off. I'd like to give the students a rather broad picture. I don't feel that the high school years are the years to specialize, even for the brightest students. To teach mathematics specifically for some vocation whether it be engineering or economics would be devastating at this level in my opinion. I don't understand first of all what the differences are at this level. I think students ought to get a broad picture of the discipline.

They usually can't get this at the university because that's where they always have some specific purpose in mind for taking math courses. I think we ought to give them a broad picture of mathematics and some of its applications, but not be so concerned that everything be approached from a utilitarian point of view. When they get to the university, they ought to be able to step into any specialized field and learn the mathematics for that field without any difficulty. At the present moment we know we do not have enough applications in the EM program. This is due to the decision we made to follow a rather logically oriented beginning. That restricts us for a while. But we have the expectation that at the end we'll have far more applications than any high school program now has, simply because students should have a real understanding of mathematics so that they can apply it. So called applications that now appear in high school textbooks are rather artificial. Students usually don't know what they're applying at all.

We're planning a series of short courses taught by outstanding mathematicians. We'll bring six of those people here a year, mainly in the applied fields, to lecture to these kids. Everything will be put on video tape or film. These will consist of 5 or 6 lectures which will be accompanied by workbooks and problem sets. The class discussions will also be on the film. We had a visiting professor give a lecture on demography which was extremely interesting and the kids really saw real application. They saw something that this professor actually developed personally to help predict the population in Western Australia universities for a period of five years. He showed the actual method he used and the students had enough mathematics so they could follow it. Now that one lecture contains for those kids, more applied mathematics of a significant nature than a full high school program called applied mathematics. The problem had relevance. It showed the students a real problem tackled by a real mathematician, and that's the kind of application that we need. It's awfully hard to find people capable of writing this level who have experiences of this type.

The superficial kind of application is where you write the problem out as a word problem. You then say that's an application of mathematics because it's expressed in words rather than symbols and seems to have some tie to reality. Students don't understand what applications are from this sort of experience. They should deal with much more significant applications. They should really see mathematics at work.

Q: *Would this be true at the elementary school level--you'd rather see applications in social science?*

A: At the elementary school level I'd just like to see kids involved in situations and call every situation an application. When a child is thinking about some little fantasy story with which we're trying to build some mathematical idea, he is really applying mathematics to his world of that moment. We try to deal with things in these stories that are more relevant in 1969 than most books do.

Q: *You use the term 'situation' more than 'application,' right? From what I gather, you think the situational aspect is reasonably important with the elementary kids?*

A: In my opinion it's the only way. It's the only way to guarantee that the student is actively involved in the development of mathematics, which to me is the most important thing. As long as what you're teaching is sound, no matter how you teach it, the kids ought to be involved in it. That means they shouldn't just be sitting back listening to somebody and watching flash cards and things like that all the time. They ought to be thinking and they have to be put in situations that make them think. And it ishard, awfully hard to write materials that guarantee this involvement in a significant way. It's even harder for us since we never worked with elementary school children before. What we know about third graders we learned here. Courses in elementary school education don't seem to be of much help for this kind of work. But, in any case, this is new for us, to write discovery kinds of materials and to do it so that it doesn't depend on a teacher. On top of the condition that we want the materials to involve the children, we put the added restriction that they are to be involved in an individual way. But I think it can be done and I think we have a lot of talent and skill on this staff to do it. If we're allowed to make mistakes and not chop up everything the minute we have a difficulty, I think we'll find lots of methods for students to find their own ways to mathematics.

Q: *What do you consider the philosophical basis of the teaching approach particularly at the elementary levels?*

A: Our philosophy is one of individualization, sound and relevant content, and students actively involved.

Q: *So you'd want a student to come up with a concept himself rather than the concept being presented?*

A: Yes, I don't like concept teaching as such where you just develop questions and answers. I'd like to see the students involved in situations from which concepts develop. Now sooner or later you need some kind of language to talk about mathematics, at least when we are ready to talk to the students about it. And so you can't say it's completely free of concept teaching.

At the higher levels, of course, there's more and more language needed. But I like to see the kids actively involved in a situation and I like them to discover things for themselves in some guided fashion, not in a haphazard way. But I don't feel that everything has to be discovered and I don't feel that it's so wrong to tell students something from time to time. The main thing we're concerned about is that the mathematics be sound and relevant and useful. So at every stage I want the mathematics to be good mathematics. There should be some rationale for anything that we do. One thing we don't want to do is a great job of teaching trivia. This is very easy to do if you're not careful.

Q: *In what ways do you consider individual differences at the elementary level?*

A: We just haven't done much in this area as yet due to lack of money and staff. We don't have the staff to really investigate the differences among students, so basically we look at kids on things like reading, I.Q., manual dexterity for use of manipulatives--things like that. As far as their learning styles and things like that, we just haven't had time to think deeply about. We do hope to eventually start attacking this problem. One attitude we take here is an empirical one. We don't start out by saying there are fifteen different learning styles and start developing materials for each style. Certainly we see some differences in these kids. A major difference is verbal skill. There might be something in terms of their social backgrounds which will turn out to be significant.

Q: *When your program is completed up through sixth grade, where would you picture that pupils would be in relationship to regular kids and regular schools? What new topics would they have covered that wouldn't be typical of any school today?*

A: Well, my hope is that their arithmetic skills will be satisfactory even though we don't overemphasize these as are usually done. We have some packages involving just pure algorithmic building but even these we think are a bit novel. We're not going to spend a lot of time on long division drill because I don't really think it's a useful skill anyway. I think the existence of the algorithm is useful, but I don't see any reason at all to worry about very much speed in doing pure arithmetic computation because that's no longer an urgent need. I wouldn't say you completely ignore speed, but I don't think speed per se is vital to the curriculum. With modern computing devices one just never has the need to have that kind of speed with arithmetic computation. So I think we are concerned that they have reasonable arithmetic skills. I don't really care if they're just average in this respect as long as they can get along in the real world. Really the whole idea of having these algorithms is so that it makes it easier to do other things. We hope that we can build skill development into the program with methods other than pure drill.

In most standard arithmetic curricula in the first six grades, there isn't very much more. Some of the modern programs have some newer topics but a lot of those are in my opinion, poorly chosen. For example, in my opinion, this business of naming numbers in different numeration systems is a total waste of time at this level. We do some stuff with Dienes' blocks which gets at positional notation, but once you get positional notation forget all the rest because it's not of much use until you get to theoretical work with number theory. We do some stuff with residue classes which you do not find in the normal curriculum. We'll do a lot of stuff with mappings and relations.

I think there's a tendency in the elementary school to make a big vocabulary course out of geometry. Set language is another thing that's overdone and largely vacuous. We do believe that set theory is very important, but we don't believe in it just as another vocabulary course. When we need the language in some situation, we develop it. We don't drill on the language; we drill on the ideas. Our elementary school kids will not be involved with a lot of pure set theoretical concepts isolated from any meaning. However, I hope they will have the facility to use some simple set language in situations that call for it. I think that the kind of geometry that we teach should depend on what you're going to do with geometry at a later stage. That's a big open question. But one thing for sure is that this program will never develop a pure synthetic Euclidian geometry course at any level. This means that almost everything on geometry in elementary school textbooks is not of very much use as far as we're concerned. It's not going to lead to the kind of geometry we want. There are four or five reasonably good geometric approaches that one could take at a later stage and we don't want to close any of those out too early for kids. In particular we feel that students should have lots of experiences with motion geometry. I want them to do a lot of paper folding, a lot of work with rotations, with reflections, use of mirror cards, that kind of stuff. Whatever you want to learn about geometric figures and so on you can learn in that context. They can get to use the words that way. We'll include a lot of space geometry--just getting familiar with space as such and maybe some of the simple vocabulary that goes with it but again not overemphasizing vocabulary. There will be some work with vector geometry. Of course, our third graders already are familiar with coordinates and ordered pairs--I think there should be a lot of that. We're just starting to think about doing some graph theory. There ought to be lots and lots of probabilistic kinds of activities going on at the elementary school. Even as early as the third grade we have three or four packages in probability already developed. Certainly by sixth grade those kids should really have been involved in lots of simulations of experiments. They should be ready to do a good probability course soon after elementary school. Probably some descriptive statistics ought to start coming in. The choice of content is wide open--lots of modern algebra, algebraic ideas. They ought to be solving equations and inequalities in contexts of a sort that are useful. They ought to be working in various operational systems like the residue classes that I mentioned before. So, by the end of the sixth grade some of them already might grasp the concept of group and field and others ought to be pretty close. The thing that we're pushing towards is lots of work with different structures and a real good feeling for the rational numbers.

Q: *Do you picture the sixth graders doing much axiomatic work?*

A: We won't do any kind of proof, certainly no axiomatics until they reach Book 1 of the Elements of Mathematics series for those that are going in that direction. We have sixth graders using it this year along with the seventh graders. The EM program is meant to start in the seventh grade because most of the students that come to us in secondary school haven't been in the elementary program. Those that are bright enough ought to be studying Book 1 sometime in the sixth grade. I think they can handle it if they're good readers. This is what it takes. Once they start the logic their picture of mathematics changes somewhat. From here on they are involved in two worlds. They learn what a proof really is. They have a formal definition of proof. They can look at something and say it is or it isn't a proof because it meets these criteria just like a computer could do it. And they also still can play in the heuristic world. Before this stage we do some informal argumentation but nothing with symbols, no symbolic logic, no truth tables until Book 1. So we keep the logic thing completely separate from the heuristic development because we think these are two distinct types of activities in mathematics that should be separated at the beginning on pedagogical grounds.

Q: *Would that be a difference in philosophy from Dienes'?*

A: Probably this view is different than any other program I know of. We use attribute blocks, but we don't call it logic and we never use the word 'prove' until we're really ready to define it. We'll say 'argue' or 'show' or 'discuss' and we leave it rather loose.

Q: *What will you include in your program?*

(Interview with B. Kaufman -- C. A. Riedesel and M. N. Suydam)

A: We have a rough topic outline for grades 4-6 that hasn't been finalized yet. This is sort of a 'hopper' from which to select content. It's really not an outline but a bunch of topics from which we think most of this program will come--a list of all possible content for the elementary school. It's sort of a smorgasboard of topics. I don't say we've covered everything but it should give an idea of what we mean by a modern elementary school program.

Q: *Some of those will be pulled then--or all of them?*

A: I doubt if any student will do all that we plan, but who knows? I don't know really how deep we can go with them. For each of these topics we must make an outline, have discussions, try the ideas on students, decide on appropriateness, etc. That's why it takes so long. We don't want any restriction at all as to what goes in that curriculum except what we feel children should have in a mathematics curriculum. We think that those kind of topics are much more useful, valuable, and exciting than just being proficient in arithmetic skills which you can't apply to anything anyway. We think that arithmetic skill will still come but we're just not going to worry ourselves silly over it. I don't think our materials could be labelled as concept formation. They are situation and activity oriented and hopefully involve the children in an active way. This is what CSMP is all about. We want the kind of content that's appropriate and works and seems to be appropriate stuff for the kids to whom it's presented on some kind of an individualized basis. We know full well that the teacher training problem for this program will be tremendous.

Q: *This is the next question that I have--what in-service training would be necessary?*

A: We have a question of what classroom model will work for this program. I don't think that the full program will work without specialists. I think sooner or later we'll just come to the conclusion that for every six classes in a school that uses this program, there will be a mathematics specialist that does all the mathematics teaching similar to the art specialist, music specialist, P.E. specialist, foreign language specialist, etc. I think that if the school system wants this kind of a program, they will have to meet the cost of an extra one or two teachers in their schools. I don't think there's any other way. Of course, this program should be ideally suited to a team teaching situation.

Q: *How do you plan to evaluate the effectiveness of the material? Are you going to get research evidence on it?*

A: We have an evaluation staff. We have a co-ordinator of evaluation. He has two assistants and is building a staff. They construct post-tests for the packages. They give attitude assessments. But I don't think we've even scratched the surface on evaluation--I'm interested mainly in formative type of evaluation which tells us how we're doing as it's being done, not how it compares with other programs. That's what we're pushing towards now. But I think it's going to take a long time for them to figure out ways to evaluate things like maturity, literacy, creativity, and those things which are most important, but difficult to define and measure even though everybody who deals in mathematics education knows what I'm talking about. I think that this will be one project in which mathematicians and evaluation people have a constant dialogue. Perhaps they will fight each other all the time, but at least there will be dialogue. This is not going to be a program that doesn't get evaluated till its completion. Evaluation will be done at all development stages.

Q: *Within the program will you be doing any research after parts are developed, in terms of sequencing or scope?*

A: It won't be so much us doing research as people who want to do research coming in here and using our facilities and working with us. We, at the moment, are not at all set up for any kind of research because our mission is one of development. But there are lots of interesting research questions that are already appearing. I would imagine before long that this program might serve as a sort of institute for research in mathematics education. We anticipate having two doctoral students soon--one from Syracuse and one from Teachers College. I think we'll get more and more of that because we've got some questions whose answers we feel are extremely important. We're interested, for example, in what effect this program is having on the reading skills of the students at the secondary level. They come with good reading skills. We want to know whether the program is improving these skills or not, whether this kind of logical development and language emphasis is affecting their other studies. There are all kinds of things that we're interested in finding out, but we just don't have the staff now to do it. It's going to have to be done by universities that send people here to do the research.

Q: *I was just noticing that you have basically two versions on multiplication in the third grade material; I'm not really sure which one would be the best starting version. You could argue for any one of the three. Would you see, for example, someone suggesting a doctoral student who is interested in exploring this kind of a problem?*

A: Oh, yes. We'll even do some 'baby research,' next year with sequencing in four pilot study classes. There's an awful lot of freedom in sequencing at this level. It gets less and less as you go on. We didn't have any research data, we just had what our eyes were telling us: we noticed that children need to have diversity at this age and so we've done a lot of blending of packages.

Q: *What do you picture, if everything goes perfectly, the major impact of the project will be?*

A: Well, I think that if we are successful and have enough time for development and research and the country's ready for it fifteen years from now or so, this could drastically change the school mathematics curriculum and the role of teachers simultaneously. It could change the methodology, presentation and content in one fell swoop. I think it could have a much higher order of magnitude of impact than any of the original new math programs had. Their content was basically the same as what they replaced, the methodology changed somewhat, maybe there was a little more emphasis on more precise ways of saying things. But if you look at the content there wasn't too much difference between that and what was there before, yet they had a major impact. So this could have even more, particularly with respect to methodology and modes of presentation, use of media and technology. This could be a big impact provided the country is ready for it at the time. Our job is to get it ready. I think we'd all be upset if nobody wanted to use what we spent a lifetime to develop, but in any case we're not salesmen. I do know that for anybody to really buy a program like this, they're going to have to see it in operation, in full demonstration. So the plan of the laboratory at the moment is to build a demonstration school or an experimental school next to our new building that will be ready in a couple of years in St. Louis. In that school not only this math program but all the other programs of the laboratory will be in operation. Then people can come and see a school in operation, not just with the materials but with whole systems approach. That's the way I think innovation should develop, not the way it has been where one builds a new lock-step to replace the old one. I think that all this building change ought to go much slower. I think there can be some minor changes in present practices which would suffice for awhile, but I don't think one ought to be making these big sweeping changes that are supposed to last for the next 200 years until the curriculum itself is changed. To me, the cancer in education is curriculum. You have to get in and cut the cancer out first. One just shouldn't innovate in a curriculum vacuum.

(Interview with B. Kaufman -- C. A. Riedesel and M. N. Suydam)

Q: *Do you see any potential dangers of improper interpretation in use of your material? In other words, are there any safeguards you'd like to suggest?*

A: Oh yes. For our materials, we just won't allow them to be used at this moment by untrained people without the proper conditions and under the right controls. We can't conceive of just handing them to a teacher and saying there they are, start using them. First, we don't have teacher guides and we don't know when we'll ever get around to that because we hope that the whole teacher problem will somehow be settled some other way than by writing teacher guides. Someday we might still have to do something like that but that's not our major concern now. But in a school which doesn't properly prepare for this kind of individualization, with special equipment and so on, and with a teacher that's not properly trained--the whole thing is going to fall on its face. We just aren't at all ready to allow that to happen at this stage. When we've finished the program, it belongs to the public: they can do with it what they want. But we can't afford to have our funds cut off because people start saying too early how poorly it works due to the fact that a proper diffusion model hadn't been developed. So it will be ready to go out out in small parts like this pilot study next year. Then the year after that, if that works, we'll probably expand a little more, with fewer controls. By the time we're through, we hope that state departments of education will have been working pretty closely with the laboratory and other laboratories to diffuse it properly. But up to that point we certainly will control it.

Project Title: Minnesota School Mathematics and Science Project (MINNEMAST)

Location: Minneapolis, Minnesota

Director: James H. Werntz

Associate Director: Roger Jones

Background

In its emphasis on the coordination of mathematics and science, MINNEMAST is unique among the curriculum projects. The curriculum has been organized around various operations and concepts from the two areas. Units are planned to provide children with activities in which learning of these concepts will occur. These activities should help pupils to solve problems using both mathematical and scientific techniques, besides encouraging recognition of new problems. A spiral development provides for continual review and extension.

MINNEMAST first developed independent mathematics and science materials for testing in various schools. These were revised, and then combined into a coordinated series of units. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Frequently, for instance, science situations and materials are used to provide an experience from which the mathematical concept will be developed. The teacher helps the children plan what they will do. Sometimes work is individual; at other times teams of children cooperate; some group instruction and discussion are also included. Demonstrations by the teacher may be used to arouse interest and raise questions to be investigated. Stories, games, and worksheets are frequently used. The development of materials to guide the teacher has been a continuing concern of the project.

INTERVIEW with ROGER JONES, Associate Director

MINNEMAST (Minnesota School Mathematics and Science) PROJECT

C. Alan Riedesel and Marilyn N. Suydam

(25 April 1969)

Q: *What are the major purposes of the project now and in the future?*

A: I would say the basic purpose is to create a coordinated mathematics and science curriculum for the elementary grades. The original intention was K through 6 but because of the fact that we are having problems with money, it's now K through 3. (We're not necessarily going to settle for that--we're looking around for other possibilities.) But anyway, I believe that we're the only national project of any size whose main goal is to coordinate both mathematics and science into one curriculum.

Q: *How did the project get started?*

A: Well, that goes way back. I'm not necessarily the best person to ask about historical questions because we go back as far as 1958 or 1959 and I've only been around here for a little less than two years. Paul Rosenbloom was the originator of the project and it was originally a mathematics program. The basic idea was that by teaching children fundamental concepts in mathematics, they would eventually learn the 'practical' aspects of math. More importantly, they would have an understanding of mathematics that children don't ordinarily get in the elementary levels. In addition to many of the ideas in the modern math program, Rosenbloom introduced three basic ideas that he saw as the basics of the program. They were the real number system, Euclidian space, and space with a measure. He saw those as the three foundation stones of the math program.

Q: *What's the philosophy of teaching? Is there any different type of teaching strategies or any type of particular learning model the teachers are supposed to use with it?*

A: I think we use several different kinds. There are regular classroom discussions and there is also individual work. The main emphasis is on trying to get the children involved with the materials and with the ideas. So there's a great deal of preparation in bringing things *to* the children--workbooks, equipment that they actually handle themselves, and things that they actually do themselves.

Q: *I just saw a lesson this afternoon that involved the teacher taking the children step-by-step--a lesson with the water clock.*

A: Okay, that's an introductory lesson. There are occasionally places where we do stop and develop something so that they can use the ideas in other things. Some of the lessons are much more discovery-oriented, with the children more or less on their own, although following a kind of route that we know they're going to follow. It's not completely free. Other lessons have the more standard didactic approach. I would say that probably 30 or 40 percent of our material is that way. In the kindergarten and the first grade there's direct presentation by the teacher of the material, and then as we get into second and third grade in particular we're trying to get more individual work--the children doing things on their own with less guidance.

Q: *How much guidance do they have in terms of what kinds of things they're to do with the materials?*

A: Well, they're usually instructed about what sorts of things they're going to be doing and some basic ideas. They may follow the book, but in a sense that's a guide too. If you mean completely free play time, there isn't as much of that as I would like to see. I think that this project in a sense is typical of many of the projects around the country today that started several years ago. The thinking has changed and become even more radical since then, and the 'old' projects are all behind the times. They are *miles* ahead of what's actually going on in the school, but not in terms of what people are thinking of doing in schools today. In the sense of a really free, open experimental school in which there's no grading and the children sort of do what they want and follow their interests and have lots of things available but no direct guidance, Minnemast as well as the other projects still have a long way to go.

Q: *They're still probably further along that way than you'd find in the schools today.*

A: Yes. Much more than what actually goes on. That's true.

Q: *What attempts do you make to handle individual differences and different levels of ability?*

A: There are optional materials which generally are for the better students. For the most part we don't provide specifically for that problem, since we aim for the 'middle' 80-90 percent of students. But the teachers have found that they can, by selecting the materials, make some things easier for the children who are having more trouble. They may use other materials for those that are doing well. We try to provide material on different levels and then expect the teachers to do some selecting on the basis of student need.

Q: *This morning they noted that the material is appropriate for 90 percent of the children. Do you think it would be closer to 60 percent of the kids of what?*

A: I don't know. It's really hard to say. I hate to say a number because I don't know. I think we're certainly aimed at the average student, possibly leaning in the direction of the brighter student.

Q: *What do you get in feedback from teachers? It's being used across almost every level and in many types of schools.*

A: Yes. The feedback varies a lot with how long the teachers have been using it. The first year they have troubles with it and after that many teachers really get sold on it almost to the point where one has to begin to get suspicious of the feedback because it's so good.

(Interview with R. Jones -- C. A. Riedesel and M. N. Suydam)

Q: *Do you find any difference in the age of the teacher or the number of years of teaching?*

A: That's been a problem to some extent. Generally, some of the older teachers find that it is more difficult for them to use the material and adapt it to their ways of doing things. That's certainly true.

Q: *I know you do quite a bit of in-service. What's your basic pattern of helping teachers to work with materials?*

A: We have teams that go out during the year and visit the teachers. They talk about the new materials that are coming out and spend time working with them. Then they visit the classrooms to help them, and give them pointers. We also have one or two orientation sessions at the beginning and in the middle of the year to provide for a general, overall view of what's going on.

Q: *Do you run some summer workshops, too?*

A: Yes. Alan Humphries, who is the director of the implementation part of the program, has tried to set up self-propagating workshops in which people come here for training. These Minnemast-trained people are then able to train teachers and other people to use the program. Some of these things have flowered, so I think it will be very useful for implementation.

Q: *There are a number of cases where you were able to export it with the help of one person going out in an area?*

A: Yes, we do visit some of these places during the course of the year and help them.

Q: *Now you're developing 'packages'?*

A: Yes. Humphries is now trying to get self-training packages going so that they don't even have to come here. They can just get a package which will introduce them to some portion of a unit or several units. Any person in the school system--an administrator or a principal, for example--would be able to administer this package to the teachers.

Q: *What are the components of it?*

(Interview with R. Jones -- C. A. Riedesel and M. N. Suydam)

A: It's going to be kind of a general multi-media type of thing, with TV tapes and filmstrips and printed materials and loops and slides and whatever else is suitable.

Q: *Do you have a target date for it?*

A: Not exactly. It's kind of an ongoing thing. It's a big project.

Q: *If it goes well, it should be very worthwhile. It seems to me we need this kind of thing in order to keep teachers everywhere abreast of things, because you can't train that many people.*

Q: *The original materials were developed with a math orientation alone. What were the changes that occurred when you switched to a math-science orientation--what happened to the math materials?*

A: Well, what happened to the math materials was more in the nature of pedagogy than it was in terms of the content of the material. I think that basically the math program still contains the components that Rosenbloom wanted, with some changes. His viewpoint was perhaps a purer viewpoint than the present one in that he wanted to do very basic and general things in mathematics and not teach from a specific unconnected viewpoint. And it turned out in using these materials that it wasn't always possible. He had, for example, a specific method of teaching multiplication so that one could see multiplication as just one among many kinds of operations that you can perform on one number to produce another number. It was a graphical method. He used this method to introduce multiplication to the children. And then when one made the transition to actual manipulations and carrying out multiplying operations, it wasn't always easy for the children. There was a great deal of difficulty. Now I've never been completely satisfied as to whether or not it was because of the difficulty the teachers had with it or because the method itself was difficult. But we did in that particular case return to a more conventional introduction of multiplication and then used the Rosenbloom technique as a kind of generalization or way of seeing it in a larger context. But the idea of introducing it that way seemed to have been very difficult for the children. So what is called the 'monolithic' approach, in which you take very basic and unified ideas and try to make everything come out of that, has had to be partially sacrificed for pedagogical reasons. Other than that, the basic changes have only occurred in attempting to integrate the mathematics with the science, but I don't believe that the math program has suffered at all.

Q: *Did the science dictate the need for changes?*

A: Not particularly. In fact, it's more often that the math sequence will dictate the science to some extent, since math seems to have more of a structure to it.

Q: *I guess in every one of the projects we've asked about their views on science-math integration. With the exception of Barbel Inhelder, almost everyone has said science either suffers or math suffers.*

A: I don't think that the science has suffered because of the fact that the math may have been more determinant in establishing the sequence. I think that they both gain from this coordination and the fact that you don't do somebody's favorite subject as well as you might have, seems to me a very small consideration.

Q: *So you feel pretty comfortable that you can develop a K-6 sequence in which math and science are integrated, and have the youngsters know more math and more science than they typically do?*

A: Yes. I always had a different viewpoint about the whole thing and a different way of seeing them, not even as separate subjects.

Teaching mathematics and science together is easier than teaching them separately. It is natural to teach mathematics with applications and illustrations from science, and to teach science when you can make use of its mathematical framework. The description of Newton's law of gravity or of the growth rate of a plant is so simple and precise in mathematical terms, that words seem cumbersome by comparison.

Similarly, the abstract idea of vectors or of the real number system can be made much clearer through physical illustrations from the sciences. This argument, supported by countless examples of the mutual reinforcement of science and mathematics, is most compelling in the classroom, as any teacher will agree. Furthermore, it is very closely related to the second reason, which is of an historical nature.

Secondly, mathematics and science have relied heavily on each other throughout the course of their common development. The very backbone of much of theoretical science has evolved, in a sense, as a branch of applied mathematics and would not exist today without it.

By contrast, the dependence of mathematics on science is not quite so explicit, for mathematics in its purest form is basically independent of science. The impact that science has made on mathematics, however, is by no means negligible. We know that many branches of mathematics would not be as advanced as they are today were it not for the impetus that has been provided by science.

These two reasons in support of a coordinated curriculum--one pedagogical, the other historical--in turn suggest a third, which is somewhat philosophical.

This third reason hints at the deeper relationship between the two subjects. It is simply that the boundaries between science and mathematics are not always very well defined. Many aspects of the two disciplines overlap so closely that they are of equal importance to mathematicians and scientists.

It may well be that thinking about math and science as distinct disciplines is not necessarily the most fruitful approach for the mathematician or the scientist, to say nothing of the layman. It seems quite reasonable to avoid making a strong distinction in the mind of a child learning the elements of these subjects. Breaking the bonds that join math and science together, for the purpose of presentation to a child, probably harms that child's appreciation and understanding of both subjects as much as it weakens the creative union between the two.

Q: *How would the youngsters that went through the program up through grade three compare with those who went through a typical science and math sequence? What more would they know?*

A: That's something we'll have to evaluate during this next year. It's really hard to say because even had the program been separately math and science, I think there are probably lots of things that would happen that would be different from what children would normally know. So you see it's even hard to understand what to compare it to.

Q: *I really wasn't saying 'is it better.' I was just asking what topics would they know that were unique?*

A: Maybe the best answer that I can give you are the topics that gain the most from the integrated approach. The two that really come to mind are measurement and functions or relationships. Those two certainly have received a lot of emphasis in very basic and extensive ways throughout the program. Both of them combine so much from science and mathematics that they are almost integrated topics in themselves and one cannot separate them. In any part of the units in which measurement is dealt with, or assignment of sizes to sets from the mathematical point of view or measurement from the scientific point of view, the concepts are very closely related to the other field. I think in those subjects in particular I would hope that the children would get a much stronger background in those particular fields. Also geometry as it relates to the sciences is another large area that I think receives an extra boost from this kind of combination. I would say that by the end of third grade we would hope that children will have a fairly good understanding of the real number system, what numbers are, and how one deals with them. There should be some understanding of space and geometrical ideas, and then some basic concepts and processes in science.

(Interview with R. Jones -- C. A. Riedesel and M. N. Suydam)

Q: *You mentioned evaluation next year. What plans do you have for evaluation?*

A: We have a team of educational psychologists that has been working with us very closely and when you mentioned feedback before I wanted to get into this. They have designed instruments for testing children specifically in terms of the goals that we have set up in our units. They administer these tests to the children and they provide us with probably the most valuable feedback that we have in the modification of the units in determining how effective they are. And that has been in the nature of a formative evaluation while we're producing materials. During this last year they are also going to get into some kind of a summative evaluation program in which they will try to summarize the whole effect of the program on children in third grade. That's a very important component of this project I think. Minnemast has another thing that some of the other projects do not. We feel much more strongly, I think, about research and evaluation. Minnemast has had this viewpoint now for quite a few years and we've been fortunate in having an excellent team of people working with us from the educational psychology department.

Q: *So evaluation is more or less planned from the beginning and included as the project moves along, rather than as an after-effect?*

A: Yes, definitely.

Q: *But as I understand it, this evaluation is mostly in terms of the units rather than anything overall.*

A: Oh, yes. Well, that's because it's been a formative evaluation so far. They've been helping us in the actual creation of the units. As the project is coming to completion in some sense, they will now try to evaluate the overall effect.

Q: *What do you picture as being the major impact of the project nationally? What things hopefully have changed because of it?*

(Interview with R. Jones -- C. A. Riedesel and M. N. Suydam)

A: Very little, very little. The major impact I would say has been on the people who worked at the project and those who have become involved with the Minnemast program in one way or another, which has not been a very large number. In some sense we have suffered from the fact that we have not yet become a commercial project. In another sense I think we have gained enormously because we have been able to steer the ship ourselves. But we have not had the impact that the other three major science projects, for example, have had, because we do not have a commercial publisher. I'm still not convinced that a commercial publisher is the thing to do although we're committed to that through the N.S.F. and we're in that process right now. But I would almost be happier if we failed to find a publisher and ended up just doing it ourselves and providing the materials, because I know the problems that people have been having with publishers. Nevertheless the fact is that we have not had as much of an influence around the country as other projects have had.

Q: *About how many people do you estimate you've reached? How many teachers and pupils have been involved?*

A: At the high point we had something like 20,000 students and 700 teachers involved in the program.

Q: *Do you run into many potential dangers in the use of the material? In other words, do you have any safeguards that you like to suggest for somebody ordering Minnemast materials?*

A: I think some kind of training or implementation or introduction is pretty essential. It's difficult to just pick the stuff up and start using it, although it's not inconceivable. But again if you talk to Alan Humphries, he'll tell you that it doesn't take a lot but it takes something to get people going. I think the safeguard that we've built is in that we haven't just indiscriminately provided the materials to people. We've always insisted that they go through some kind of training program or that we visit them or hold a workshop or something like that. We have controlled distribution of the materials through our own centers and these 20,000 students were all in our own centers. Some of them were self-supported but they all had people who were familiar with the materials through a workshop or an orientation program.

Q: *You mentioned earlier that you'd like to complete the program for grades 4 through 6. Is this the major plan or major hope of the future now?*

A: Well, I'm also interested in seeing some kind of a more flexible program developing along the lines that we were talking about before--an experimental school system. I think that's the direction I would like to see things going. And curriculum development would be a component of that unless you think of curriculum development in a larger sense in which it just means a whole school. So I would hope that as some component of a future version of this or some other project which might develop from it, we would be able to finish the math-science program for grades 4 through 6. We're about halfway through third grade now or three-quarters of the way in preliminary versions, and by the end of next June we should have final versions similar to the regular printed units yes.

Q: *Do you have math materials that were developed prior to the change in emphasis?*

A: Yes, we do. Those materials still exist although we're not reprinting them. But we have math materials through fourth grade and even some fifth, I think.

Q: *How important do you feel the applications or the social phase of mathematics is? Certain mathematics programs of recent years switched to more abstract programs, less application-oriented. In the science programs many are more experimentally but yet less application-oriented. Is this the wave of the future or do you think that there will be a swing toward greater tying in with the world of the child?*

A: I think somehow you have to make it meaningful to the children, but I'm not sure that that necessarily means practical applications in the sense that we often think of. I think the Minnemast program has always made an effort to teach general principles, but always somehow in terms of simple, applicable, understandable ideas in the world of the child. I think of this question that you're asking in a much broader sense about how science and mathematics should be presented in general. I'm not sure how applicable this is specifically to elementary school, but I'm not at all convinced that the answer to the 'relevance' problem is really one of applicability and utility. I feel, because I teach physics at the university, that if all that I tried to do in a physics course is show people the connection with TV sets and toasters and refrigerators, in some sense it would all go for naught. There is much in physics and mathematics and many sciences that not only has been of enormous practical value but also has changed people's ways of thinking. A change in philosophy or viewpoint, it seems to me, is an important thing for students to appreciate. It's a difficult thing to do--to show how physics or mathematics or anything has affected human beings' lives in this very deep sense, but certainly that's as relevant as anything can be if you can get it across. So although I believe it is a question of making it meaningful to kids and putting it on their level, I'm not sure that that necessarily means simple,practical and obvious applications. Somehow you have to make philosophy more concrete for people.

Q: *How many materials have already been produced?*

A: We have kindergarten, first, and second grade materials. Kindergarten and first grade are complete, second grade is almost complete. We're just printing the last few units of the second grade, and the third grade materials are still in the preliminary version. We have equipment kits for again K, 1 and 2. The third grade again is in the formative stage.

PRESS CONFERENCE: JEAN PIAGET AND BARBEL INHELDER

Los Angeles, California

(7 February 1969)

Q: *Dr. Inhelder, could you briefly explain what your work is about? Then perhaps the reporters could ask questions of Dr. Piaget through you.*

A: It's a very free interpretation I'm giving you now. Our main interest is the study of the relationship between memory, particularly recall or evocation memory, and the development of intelligence in children. We are asking ourselves if memory is linked with cognitive development or if memory is a kind of passive reproduction of perception. To study this kind of relationship we selected twenty-five different kinds of experiments which we carried out on children aged between four and eleven. For instance, there is one very simple experiment, which I shall talk about this evening. The children are shown ten sticks arranged in a series from the shortest to the longest. They are asked to look at them so as to be able to remember them later. After a short interval (up to one week) and again after a long interval (between six to eight months) we ask the children first to indicate by gesture what they remember and then to make a drawing. We noticed that the small children don't remember the seriation, but assimilate what they have seen to their cognitive skills, to their cognitive understanding of seriation. They do the same thing in drawing as they do when we ask them to carry out the action of ordering sticks from the shortest to the longest. For instance, they put one short, one long, one short, one long, etc.; or all the short ones together and all the long ones together. Slightly older children, however, draw groups of short, medium-sized and long sticks. They really believe that this is what they were shown. Our six-seven year-olds' memory of the seriation is, however, much more accurate. So we feel that they must have assimilated and tried to understand what they were shown and are recalling the result of their own understanding. What is even more interesting is that when we ask a child, let's say a three- or four-year-old, six months later to recall what he has seen and to make drawings, we find that there is a kind of evolution going on in his recollection, in his coding system. It is this coding system which we think is of the utmost importance. The encoding and decoding system is dependent upon the code, which is itself closely related to cognitive development. It is constantly changing and it is this factor which is so important. Our main problem is thus the relationship between memory and intelligence during development and not rote memory.

Q: *Are you saying that the child automatically develops this cognitive feeling for what he is learning by rote as he goes through the rote maneuvers?*

A: Automatically is perhaps not the right word because the child is always active. During the six-month interval between the presentation of the model and the recall he does a lot of things; his cognitive development is continuous. The memory processes are, of course, very closely linked to cognitive activity as a whole through the child's everyday activity; nothing happens completely automatically. If a child were to do absolutely nothing then there is every reason to believe that nothing at all would happen. It's through his cognitive activity and his own manipulation of things that a child's memory becomes better and better. But that's just one example of what we might call "straightforward" development; sometimes there are conflicting situations and regressions in memory evocation may result.

Q: *Then you make a distinction between the depositing of the impressions in the brain, or whatever you call it, and the recall at any particular time?*

A: Yes.

Q: *Then if you take an impression that's supposed to be registered in the brain, but later on the child or adult tries to recall this different kind of activity, then there's an interpretation?*

A: As a result of the advances in neurological and biomolecular thinking, we no longer believe in simple memory traces. The whole nervous system is also changing all the time, although there is some continuity in brain growth. It is not a question of there being a simple imprint which the subject can produce in its original form whenever he so desires.

Q: *But do the deposits or imprints themselves change? After a particular molecular has taken place in the proteins, they do not change afterwards, do they? I mean if you see something--a flash of light --and it makes some kind of an impression, then that still remains the same although later on you try to interpret what you saw--you add or subtract something of that impression. Is that what happens?*

A: It's one possible solution. We can't decide the issue from the behavioral point of view, but, for instance, Professor Weiss from the Rockefeller University shows through movies of the nervous flow that cell movement is going on and changing all the time. This biological point of view comes closer to our behavioral interpretation that memory is continuous reconstruction.

Q: *How does this clarify or agree with Freud's theories of repression of memories?*

A: Piaget is a little skeptical about childhood memory and likes to give an example from his own early life. He thought he remembered very well that when he was on vacation in Paris and still a baby in a pram, his nurse took him out on the Champs Elysee. Someone hit him. The nurse struggled with the man who had hit him and was slightly injured as a result. The family believed the nurse's version of the incident and gave her a gold watch. Many, many years later, the nurse had a religious conversion and confessed that the whole thing had been pure invention. Piaget thinks that he must have heard this story when he was four or five years old and reconstructed it as his own 'memory'. This makes him doubt the authenticity of infant memories. In fact, it was a memory of a memory and a second degree memory sometimes transforms the earlier one. We are, we think, perhaps justified in being a little skeptical about the contention that early childhood memories are conserved in the strict sense of the term without any reconstruction. In fact reconstructions go on at every age and we are always simplifying, modifying or adding to our memories.

Q: *We talked about memory in young children. We talked about the sense of their learning new experiences. Is the coding that you're talking about similar to the use of associations to encourage memory? Is that a more simple version of what is happening to the youngsters?*

A: I think you have made two separate points: firstly, how does the child learn something new and understand new experiences? Secondly, how does he recall what he has done, seen or heard? These are, of course, two quite different things. We saw in our learning experiments that the child is also assimilating new facts, new perceptions of all kinds and, if we consider language, new words to his own mental schemes, that is to his own mental activities. He also transforms the information, that is the input, so as to fit it into his own system, but at the same time this information is itself modifying the system, so that it's a whole process of reconstruction.

Q: *It's an adaptive process?*

A: Yes, it's an adaptive process. We have assimilation and accommodation of the information to the new environment and the environment to the input.

Q: *I think the gentleman is also asking about the associationist position with respect to coding. Would you speak about that?*

A: Piaget doesn't deny that associations are going on all the time, but he thinks that is an artificial fragmentation of a much more general process. Take, for instance, the famous example of Pavlov's dog. The dog salivates not only in contact with food, but in response to other stimuli originally connected with food; it is thus not only the associative mechanisms that are involved, but also the dog's needs and their satisfaction, in fact a whole system of biological and associationist links. To take only associations into account would be, as I have just said, an artificial fragmentation of the whole process. And so we think, moving just one step further to education, that if you only try to associate, establishing links without meanings which the child already possesses, that is without giving a meaning to what the child has to learn, then the whole thing becomes so artificial that it becomes senseless for the child.

Q: *This is the way we are taught, at least in American schools, to learn reading, writing, and arithmetic by rote. Do you have any suggestions for implementing a more significant curriculum to help the total learning process?*

A: The most important thing is not to destroy the child's curiosity as regards problem-solving and to give him enough opportunities, plus of course some guidance, to solve problems and rediscover scientific discoveries by himself. Of course, he needs some guidance. You You have to break large problems into small ones, but you must not do everything for him, otherwise he would become a passive and not a creative individual. That's probably the most important thing.

Q: *He should make use of the memory to create something rather than just become a passive encyclopedia?*

A: Yes, and it's important to develop your own coding system. We are so overwhelmed by all the information that comes in that we have to build up a coding system. Of course, the most rational and economic ones will be the best.

Q: *You might include also the need to match the material and the situation to the child's level of mental development. This not only creates a situation in which he can be self-regulated and active, but also creates the possibility of him assimilating the material into the total schema at that time, and not just build up artificial associations that have no significance in terms of the total mental level of development.*

Q: *Given that set of circumstances you just described now, would this coding system develop automatically, unconsciously within the child? Would he do this himself?*

A: **Unconsciously, yes, of course. The child is not conscious about the whole apparatus, his information-digesting system. It's not just a problem of maturation. It's really the outcome of an active self-regulated system, if you think in biological terms. So it's going further and has a direction. It's not just a chance procedure. There are quite definite necessary steps. There's also a kind of serial order in development which we must determine and we must find out how the child is able to assimilate new information so that we can adapt the curriculum and educational material.**

Q: *Dr. Inhelder, what is the interpretation of consciousness and unconsciousness in your point of view? We always use the words unconscious, conscious--just precisely what's the scientific meaning as you use that in your work?*

A: **Piaget makes the point that the frontier between the conscious and the unconscious phenomena is constantly moving.**

Q: *It's a concept of a spectrum?*

A: **Yes, and in cognitive problems, not only in emotional problems as Freud showed us. The underlying structures of thought are unconscious. We're unconscious of our logical system.**

Q: *Does that simply mean a lower degree of consciousness when you say unconscious, or does it actually differentiate between the unconscious and the conscious?*

A: It is a question of degree. But if there are cognitive conflicts and we have to adapt ourselves to environment and solve the problems, then we become conscious of our whole mental activity, that is, as Piaget says, it is the conflict between the subject's schemes and the environment that awakens our consciousness.

Q: *Do you believe that your findings apply to children the world over for different socio-economic groups?*

A: One of my co-workers went to Algeria and studied unschooled children. In some of the fields she tackled she found that these unschooled children acquired the concept of conservation more slowly than their Genevan counterparts but through a parallel process. In others, she found that sometimes the younger children in the unschooled environment seemed to do much better at an earlier age, but that later on they temporarily 'lost' their conservation. We must study these processes in all the different fields of knowledge in different environments. There are also some experiments using our procedures going on in the United States, which show, for instance in language development, that some of the underprivileged children, who as you know have some language difficulties, lack many of the lexical acquisitions of their more privileged counterparts. As regards the basic structural (grammatical) aspect, both groups of children are more or less at the same level. It is the syntactic level which is closely related to that of general cognitive development.

Q: *Would this indicate the need for some kind of caution in using your findings and applying them to some other groups where possibly experiments haven't been conducted in their typical environment?*

A: Yes. You have to do the experiments first, but it is also a problem of methodology. If you are really trying to interview children, you must adapt your approach to their own language and habits, then and only then, can you bring out their underlying structures. So it's not just a question of performance. There are always great differences in performance levels, but the underlying structures are much closer to each other in the different environments than was formerly thought. The difference in the underlying competence and structure is the important thing. Professor Furth at Catholic University in Washington has used our procedures with deaf children. He has not found a very great difference in these children's reasoning processes. Of course, he had to adapt the whole procedure to take account of their deafness.

Q: *But would you say that there are some kinds of fundamental things that you found, that you would assume would apply to probably all children with various variations?*

A: There can be accelerations, decelerations, more rapid or slower development, but the sequence for the most fundamental things seems to be analagous.

Q: *David Elkind has quoted Dr. Piaget as saying that he has no desire to recruit disciples or found a school. I'm rather curious about this. Is there any formal structure in being that teaches or utilizes Piagetian philosophy? I've read that Montessori in some aspects is very comparable; in fact I've read that Montessori is the pragmatist while Dr. Piaget is the theoretician. These all involve, you can see, looking for a direct pragmatic application.*

A: Pragmatic applications to education from Piaget's psychological findings should be made by educationalists. He himself does not feel qualified to extract and give to educationalists a deductive system derived from his own system of thought. So he's very careful about making such deductions and giving advice to people in this closely related, but still different, discipline. Piaget has always been against a dogmatic closed system and does not want to be the founder of such a system. He is very happy, however, that educationalists themselves have started to apply his findings to their curriculum studies.

Q: *And I think the problem arises from the fact that he has emphasized that there is a relationship between his theoretical system and the methodological requirements for investigating some of the problems that the theory poses. So at times he has been critical of either those who have confirmed his findings or those who say 'Piaget is all wet.' Not because he doesn't appreciate the intellectual effort, but rather because he realizes it's necessary, in order to test a theory you have to be able to understand the theory sufficiently well to incorporate in your test the methodology that is appropriate. I think that's one thing; and I think the other is you can never avoid generating a population of disciples if you have creative ideas that seem to work. And that's not his fault.*

Q: *What questions do you hope to be probing in your investigations or are you continuing with intelligence and memory?*

A: Our Genevan research is continuous. At the moment Piaget is involved in problems of causality, how to understand the physical world. With a team of researchers, I am studying the passage and its dynamics from one stage to another. It is not enough to study the stages and the underlying structures, we must also determine the dynamic processes involved. For instance, a great many psycholinguistic studies are being carried out. My colleague, Dr. Sinclair, is studying the role of language in cognitive development. Miss Bovet is carrying out intercultural studies. So we are trying to go further and more deeply into the study of the whole cognitive development than has ever been done before. Then there are the educationalists who are trying, in collaboration with us, to apply our findings to mathematics and physics teaching. Together with psychiatrists, we are studying children who find adaptation difficult: psychotic children, feeble-minded children, retarded children, children with reading disabilities and so on. We are trying to adapt our material and our procedures to the clinical investigation of these children. This is one of the most important aspects of our research.

Q: *We've had, in the past ten or twenty years, a knowledge explosion--tremendous amounts of new knowledge with more and more to understand.*

A: Piaget has discovered that the young child's underlying structures closely resemble some fundamental principles of modern mathematics. We are trying to determine if this is also the case for modern physics. If this is so, then we shall have the best possible conditions for the acceleration of the child's understanding of the new information. It is very important not just to accelerate everything, because that is a waste of time and the child becomes too passive. A child should become an active, creative person so you have to take into account his own way of assimilating things. What is important in Piaget's system is that he is also in close contact with modern scientists, physicists, mathematicians and logicians. So he's not only--I'm going a step further--he's not only observing the children, but is also involved in new trends in modern science and is able to ask the help of the scientists themselves.

Summaries of Other Developmental Projects: Long-Term

The projects listed in this section all deal with elementary school mathematics curriculum improvement. The purpose of each project is briefly noted. Further information on each can be secured from the Report of the International Clearinghouse on Science and Mathematics Curricular Developments, edited by J. David Lockard (available from the Science Teaching Center at the University of Maryland).

Project Title: Canary Islands Mathematics Project (CIMP)

Location: Las Palmas de Gran Canaria, Canary Islands

Director: Julian B. Caparros Morata

Purpose: The experiment is to determine possible ways to improve Spanish teaching of mathematics, with investigation of learning against teaching, and experimentation against verbalization. Methods of instruction used include independent study, laboratory investigations, lectures, seminars, discussion groups and work cards.

Project Title: Experimental Project on Teaching of Science and Mathematics at the Middle School Stage.

Location: New Delhi, India

Director: R. N. Rai

Purpose: The project aims to achieve higher standards of science and mathematics instruction in grades 6-8. New texts, teachers' guides, laboratory experiences and equipment are being developed for complete package programs to be used with teachers for short-term or refresher training and for students.

Project Title: Frankfurter Project

Location: Frankfurt, West Germany

Director: Heinrich Bauersfeld

Purpose: Mathematics for k-4 is being modernized through development of new student and teacher materials and methods. The cognitive processes of thought in relation to language and symbols is being investigated for the non-mathematical effects of mathematics instruction.

Project Title: Grand Rapids Mathematics Laboratory Project

Location: Grand Rapids, Michigan

Director: Lauren G. Woodby

Purpose: The project involved the improvement of mathematics teaching, especially in inner city middle schools, by training teachers in the use of laboratory-discovery approach, as a follow-up of the Cleveland Mathematics Laboratory Pilot Project.

Project Title: The Greater Cleveland Mathematics Program of the Educational Research Council of America (GCMP)

Location: Cleveland, Ohio

Director: John F. Mehegan

Purpose: The major purpose was to improve k-12 mathematics instruction by development of a mathematically correct and pedagogically sound program that is comprehensive and sequential.

Project Title: Individual Mathematics Program (IMP)

Location: Hawthorne, Victoria, Australia

Director: M. L. Clark

Purpose: Modern mathematics text, assignment materials, and teaching materials that would allow individual progress are being developed. The materials are based on a national outline of course content for grades one through six and include self-directed and teacher-directed instruction.

Project Title: Introduction of Some Concepts in Mathematics to Primary and Secondary Schools

Location: Trinidad, West Indies

Director: Sair Ali Shan

Purpose: Investigation of children's reactions to certain mathematical topics using specified psychological principals is being carried out. Topics for ages 5 to 11 include modern geometry, sets, and number systems with different bases. Psychological principles include proceeding from concrete to abstract, Piaget's principles of conservation and reversibility, and Wertheimer's concept of structured thinking. Initial purpose is to help teachers and children be knowledgable of current trends in mathematics education.

Project Title: Nuffield Mathematics Teaching Project

Location: London, England

Director: Geoffrey Matthews

Purpose: Materials for teachers providing a contemporary approach to mathematics for children, age 5-13, are being developed. Teachers discuss and develop the materials for content mastery, process acquisition, attitudinal changes, encouragement of independent thought and experimentation.

Project Title: Patterns in Arithmetic (PIA)

Location: Madison, Wisconsin

Director: Henry Van Engen

Purpose: Modernizing and re-orienting mathematics programs is proposed through provision of a complete arithmetic course (grades 1-6) and televised in-service work for teachers.

Project Title: Science Education Center, University of the Philippines (SEC)

Location: Diliman, Quezon City, Philippines

Director: Dolores F. Hernandez

Purpose: Various purposes include the improvement of locally produced science and mathematics textbooks, development of a curriculum model, preparation of curriculum materials for grade 1-10 students and teachers, and promotion and increase of knowledge, innovation, imaginative inquiry and independent learning.

Project Title: Survey of Recent East European Literature in School and College Mathematics

Location: Chicago, Illinois

Director: Alfred L. Putnam

Purpose: Materials and information in mathematics and mathematic education from East European sources are being translated and made available for teachers and students. Original materials were written in Russian.

Additional information on the following projects is also available in the Report of the International Clearinghouse:

Boston College Mathematics Institute
California Mathematics Improvement Programs
Contemporary School Mathematics (England)
Frederick County Quality Improvement Program for Mathematics (Maryland)
Experiment on Systematic Teaching of Probability (Rumania)
Individualized Mathematics Instruction (Sweden)
Mathematics and the Direct Training of Basic Conceptual Skills (England)
Midlands Mathematics Experiment (England)
Pennsylvania Retrieval of Information in Mathematics Education
The School Mathematics Project (England)
The Shropshire Mathematics Experiment (England)
University of Maryland Mathematics Project

Summaries of Other Developmental Projects: Short Term

This section includes primarily projects in elementary school mathematics which were funded under Title III of the Elementary and Secondary Education Act of 1965. It is as complete as possible; however, there are undoubtedly projects for which no information was secured.

The summary for each project indicates the primary purpose or purposes, a brief description of the procedures used, and a brief statement of the most pertinent and valid results, findings, conclusions, or implications of the project. Reports of all of these should eventually become available through the Educational Resources Information Center (ERIC).

Project Title: Computer-Assisted Instruction Laboratory in Mathematics and Science (ES 000 770)

Location: Kansas City, Missouri

Director: Joe R. Dean, Jr.

Purpose: The feasibility of an IBM 1500 computer system with a unified math-science curriculum was being studied.

Procedure: Each of 650 eighth grade students spent an average of 45 minutes per week at a computer terminal. Records were kept to determine the degree of individualization of instruction and the effect on both students and teachers of innovative, problem-solving activities.

Results: At the time of the report, the program had been in operation only four months; no evaluation was included.

Project Title: Computer-Assisted Instruction in Mathematics

Location: McComb, Mississippi

Director: Julien D. Prince, McComb Municipal Separate School District

Purpose: In order to improve the students' scores on the Stanford Achievement Test in elementary school mathematics, the district sent twenty teachers to a summer workshop at Stanford University to learn to operate and manipulate Computer-Assisted Instruction material.

Procedure: The twenty teachers participated in the summer workshop, returned to their school district, and with a consultant from Stanford University, were then consultants to an additional eighty-one teachers. A trial program was initiated and evaluated.

Results: Results of the evaluation show that achievement gains ranged from three months in grade four to one year in grade one. Tentative results indicate that this CAI drill to be more effective than conventional methods.

Project Title: The Development of Four Model Elementary Programs for Teaching Mathematics with Implications for Other Instructional Areas (ES 001 875)

Location: Greensboro, North Carolina

Director: Sadie M. Moser, Greensboro Public Schools

Purpose: The project was conceived to bridge the gap between current educational theory and practice, through demonstrating several organizational patterns for an effective mathematics program. Its two major objectives were (1) to improve instruction in mathematics and other subject areas for all children in the upper elementary grades and (2) to provide a climate for exploration and experimentation with innovative approaches.

Procedure: Four schools were designated to demonstrate model programs using different organizational patterns: (1) self-contained classroom, (2) departmentalization, (3) team teaching, and (4) specialist teachers. All teachers participated in a workshop emphasizing modern mathematics concepts, individualizing instruction, and the use of materials. Specific workshops were also held for teachers in the team teaching and departmentalized model schools.

Results: Data from questionnaires and observations indicated:

(1) parents and teachers supported the innovative programs, with teachers being more supportive than parents of the team teaching and specialist teacher programs.
(2) interest of teachers in proving mathematics instruction increased.
(3) students in general have a favorable attitude toward mathematics.
(4) use of test results for diagnosis increased.

Mean scores in mathematics on standardized achievement tests were highest for the self-contained classroom approach. Poorest results were consistently produced by the specialist teacher approach.

Project Title: Elementary School Developmental Mathematics Programs (ES 001 692)

Location: Woodbury County, Iowa

Director: David Grindbery, Sioux City

Purpose: The purpose of the program was to develop teacher skill in individualizing instruction within the framework of the existing self-contained classroom.

Procedure: A summer workshop involving one teacher from each of thirty schools conducted parents, students, teachers and various experts was conducted to help the participating teachers develop techniques for individualizing instruction. During the school year a bi-monthly conference was held to evaluate results. During the second summer another workshop was held to help the teacher become more sensitive to the various differences among students. A third workshop was planned.

Results: The most significant result is the teachers' attitudes toward teaching, students, and parents: an attitude of co-operation. Additionally teachers profess a greater understanding of their subject and are better able to use supplementary material that was available in their area.

Project Title: Improving Teaching Strategies Through Video Taped Classroom Demonstrations

Location: Beaumont, Texas

Director: Billy N. Pope, South Park Independent School District

Purpose: Ideas in content, methods, and use of multi-media are to be disseminated by video taped classroom demonstrations, for in-service and pre-service teacher education.

Procedure: The demonstration teacher for a given lesson prepared the lesson plan, evaluated and revised it in consultation with a staff drawn from thirty-four sources, taped a practice session, evaluated and revised it with the staff, and finally taught a "live" class of fifteen unrehearsed students. The final tape was reviewed by a panel to select the best for kinescoping.

Results: The kinescopes have had statewide and national appeal and have been in constant demand. They are used in twenty-five institutions of higher education to expand the experience of pre-service teachers.

Project Title: Individual Computer Aided Instruction

Location: Paintsville, Kentucky

Director: Edwin R. Jones, Paintsville

Purpose: In the initial stage, the purpose was to demonstrate and evaluate the effect of Computer-Assisted Instruction in the area.

Procedure: During the winter of 1968, all teachers attended a workshop at Morehead State University in conjunction with Stanford University to learn procedures and programs for the Computer-Assisted Instruction terminals of which there are thirty, located in ten schools throughout eight districts in the area. The programs to be initiated were the Stanford Logic program and the drill and practice program for adult education.

Results: (No results are yet available.)

Project Title: Individually Prescribed Instruction Project Edinn-Project for Educational Innovation (ES 000 834)

Location: Monterey, California

Director: Edwinn C. Coffin, Superintendent of Schools

Purpose: This is a report of the field test of the Individually Prescribed Instruction developed at the University of Pittsburgh.

Procedure: A summer workshop for involved teachers was conducted to acquaint them with the equipment, programs, and the prescription method for individualizing instruction by finding the most advantageous programs for each student. During the school year the teachers met weekly to discuss the programs and the prescriptions for their students.

Results: The results indicate that the students involved show more interest in school and mathematics, seem to show more depth of understanding in mathematics, and that individualization becomes a method for independence and generalizes to other academic areas. However, there are indications that the program is less successful with slow learners, and that for several reasons the results of standardized tests are disappointing.

Project Title: Mathematics Information System Satellite Center

Location: Harrisburg and Greensburg, Pennsylvania

Director: Emanuel Berger and Doris Creswell, Department of Public Instruction; Frank Ziaukus, Director, Mathematics Information System Satellite Center, Greensburg.

Purpose: A computer system storing textbook analysis, research evaluation, and analysis of achievement tests is to be made available to communities and persons making decisions about elementary school mathematics curriculum.

Procedure: Using the *Content Authority List* and the *Behavior Authority List*, the Department of Public Instruction of Pennsylvania analyzed all of the textbooks of elementary mathematics in print to create an open-ended file in a computer program known as PRIMES. A similar list was used to create files for research and tests. Computer terminals are to be established in each administrative unit of the state, the first of which was established at Greensburg. During the first two weeks of the Greensburg installation a workshop was conducted to instruct personnel and other users in the operation of the center. As a general service, users may address questions to the center by telephone, in writing, or in person, and a specialist at the center will prepare the answer.

Results: (No results are yet available.)

Project Title: A Multi-Discipline Educational Center and Services Designed for the Diffusion of Emerging Instructional Techniques and Curriculum Patterns in Individualizing Pupil Teaching and Learning.

Location: Bowling Green, Kentucky

Director: O. A. Mattei, McNeill Elementary School

Purpose: The McNeill school serves as an experimental educational center which provides upgraded, unpaced educational experience through the first eight grades.

Procedure: The center provides a mathematics team with teachers to assist each child in his program and to suggest sequences of programs and counselors to evaluate progress. The center provides in-service education to teachers from surrounding districts. The center keeps parents informed about the students' objectives and progress toward those objectives. The center also conducts a summer workshop for teachers directly involved.

Results: Although all of the results are not yet available, the center librarian reports that the number of books that the center students use compared to non-center students is voluminous.

Project Title: New Shoreham Tele-Lecture Math Project

Location: New Shoreham, Rhode Island

Director: Arthur McMahan, State Department of Education

Purpose: This project was initiated to provide elementary school consultation and instruction in mathematics to an isolated, island community off the coast of Rhode Island which wanted to modernize its elementary school mathematics program.

Procedure: Two-way communication between the community and Rhode Island College was established by means of an amplified telephone and electro-writer system.

Results: The system provided the necessary instruction for the teachers and pupils in the elementary school as well as providing the impetus for a weekly meeting between the teachers and parents as the parents became interested and involved in the program. Furthermore, the teachers are stimulated to accept more responsibility for their own expanded education and enrichment.

Project Title: Program Development and In-service Training for Improvement of Curriculum Organization and Instruction in Carteret County North Carolina Schools.

Location: Carteret County, North Carolina

Director: Douglas M. Jones, Beaufort, North Carolina

Purpose: It is proposed that depth evaluation of curriculum and teacher preparation and the resulting changes will improve the achievement of the pupils in the schools affected.

Procedure: A consulting team, composed of forty teachers and principals, and supervisors, will conduct a depth evaluation of the elementary school mathematics curriculum, using a specially designed evaluation instrument from the State Course of Study, and pretest the teachers and pupil involved.

Results: (No results are yet available.)

Project Title: A Program of Teacher Re-Education for Curriculum Development

Location: Reidsville, North Carolina

Director: J. W. Knight, Reidsville

Purpose: It is hoped that through summer workshops, teachers will be able to diagnose elementary school mathematics difficulties and prescribe corrective individualized instruction.

Procedure: Three summer workshops, conducted by specialists from The University of North Carolina, will be held in diagnosing, learning and individualizing instruction theory, and evaluation of various techniques. During the school years, experienced consultants from Norwalk, Connecticut, will help the teachers translate theory into practice and become consultants to each other.

Results: (No results are yet available.)

Project Title: Planning and Pilot Implementation of a Computer Based Instructional Program (ES 001 917)

Location: New York City

Director: Cornelius Butler, New York City

Purpose: This project will determine the feasibility of conducting a computer-based drill and practice instructional program in a large city school system. The value of CAI in terms of pupil achievement, field testing of the specific program, and gathering of data about instruction and learning via CAI are specific objectives.

Procedure: For the initial period, the schools were connected via telephone line with the Stanford University computer, accessing the arithmetic drill and practice developed under the direction of Partick Suppes. During the next stage, the district's own computer was used for storage and transmission of the program.

In-service training of teachers and parent orientation are components of the program.

Results: (No results yet available.)

Project Title: Prototype: Leadership Training Program Demonstrated Through a Leadership Training Program in Elementary School Mathematics Education (ES 001 600)

Location: Brookline, Massachusetts

Director: Clarence W. Bennett, Boston State College

Purpose: Teams of educators for nine widely separated communities were exposed to new elementary mathematics curricula, materials, and personnel to (1) help them acquire knowledge of recent trends and developments, and (2) plan for curriculum improvement on the local level. It was hoped that these educators would act as leaders for accelerated curricular reform in their own communities, adapting to specific needs and perhaps modifying the plan for other subjects and grade levels.

Procedure: A six-week summer program provided the teams with basic courses in number systems and geometry, and lectures on learning theory and its implications. Current materials and trends were explored and discussed. Curriculum and guideline materials and recommendations were prepared by the teams. Several kinescopes were developed.

Results: Reports of participating teams described revisions and improvements in the curriculum guides in their communities, and indicated that the leadership training purpose was achieved.

Project Title: First Grade Children's Concept of Addition of Natural Numbers

Location: The University of Wisconsin - Madison

The following reports have been published by the Center:

I. **Report Title:** First Grade Children's Concept of Addition of Natural Numbers

Authors: Henry Van Engen and Leslie P. Steffe

Date: 1966

Summary:

One hundred first grade pupils from five schools serving a middle-class population were individually tested, both orally and on a written test, on four concept-of-addition items.

Analysis of variance showed no significant differences between boys and girls, among pupils from the five schools, or in complexity.

A significant relationship was found between I.Q. and ability to abstract the concept of addition of whole numbers from the physical situation.

The scores on the written test indicated that nearly all of the children had memorized the arithmetic facts, yet only half of the children correctly stated no preference in the physical situation for the smaller or larger amount.

The interpretation suggested was that children at the first grade level have not abstracted the concept of addition from physical situations but have memorized the addition facts. Implications for arithmetic instruction were discussed and researchable questions were raised.

II. **Report Title:** The Performance of First Grade Children in Four Levels of Conservation of Numerousness and Three I.Q. Groups When Solving Arithmetic Addition Problems

Author: Leslie P. Steffe

Date: December 1966

Summary:

A test of conservation of numerousness was developed to separate first grade children into four levels to study their relative performance when solving arithmetic addition problems. Within each level the children were placed in I.Q. groups (78-100, 101-113, 114-140), so that it was possible to study the relative performances of the I.Q. groups also. Two variables of interest were: (1) presence of described transformation vs. no described transformation of sets in an addition problem; (2) the presence of (a) physical aids, (b) pictorial aids, and (c) no aids when solving addition problems. A test of addition facts was conducted as well as a correlation study of the scores when solving addition problems and when responding to addition facts.

Excellent prediction of the relative success in solving addition problems and learning addition facts can be made for children entering the first grade as all groups performed at significantly different levels according to comparative level. The problems with a described transformation and those with accompanying aids were significantly easier, and all correlations were significant ($p < .01$).

Results also indicate that there are three categories of children for which it can be justified that the types of experience presently provided produce different results with respect to solving addition problems; research is needed to determine the most suitable curriculum for each category. Drill procedure on addition facts is quite ineffective for those children who experience difficulties in solving problems.

III. Report Title: The Effects of Two Variables on the Problem-Solving Abilities of First Grade Children

Author: Leslie P. Steffe

Date: March 1967

Summary:

Ninety first-grade children were randomly selected from three school buildings, each of which used a different arithmetic program. These children were individually tested on twenty addition problems which were read to them by one experimenter. Forty-five of the children received problems which involved an existential quantifier and forty-five received problems which involved no quantifier. Ten of the twenty problems each child received were problems in which the names that described the three sets involved were all different, while in the remaining ten problems, these names were the same.

Analysis of variance showed no significant difference between the problems with an existential quantifier vs. those with no existential quantifier, and no significant difference among the pupils from the three schools. A significant difference did occur, however, between the problems which involved common names vs. those which involved different names.

IV. Report Title: The Effects of Selected Experiences on the Ability of Kindergarten and First-Grade Children to Conserve Numerousness

Authors: E. Harold Harper and Leslie P. Steffe

Date: February 1968

Summary:

This study was designed to test the effects of a sequence of 12 lessons on the ability of kindergarten and first-grade children to recognize and conserve numerousness. The sequence of lessons involved the following concepts: (1) one-to-one correspondence, (2) perceptual rearrangement, (3) as many as, (4) more than, (5) fewer than, (6) addition, and (7) subtraction. Two pretests were administered to the children in each grade level, the Lorge-Thorndike Intelligence Test and a test for numerousness. One posttest was administered to each grade level, the test of numerousness. A separate analysis of covariance was performed at each grade level; covariates were the scores from the pretests. Significant differences were observed between the adjusted means of the experimental and control groups at the kindergarten level in favor of the experimental group, even though both groups had gained.

It was concluded that, for the kindergartners, the lessons were successful in enhancing the children's ability to conserve numerousness.

V. Report Title: The Development of the Concepts of Ratio and Fraction in the Fourth, Fifth, and Sixth Years of the Elementary School

Authors: Leslie P. Steffe and Robert B. Parr

Date: March 1968

Summary:

Six tests, four on the pictorial level and two on the symbolic level, were constructed to measure performance in three ability groups on problems classified as ratios or fractions. Two variables were of interest in the four tests on a pictorial level: (1) "equal" ratio or "equal" fraction situations, and (2) missing numerator or missing denominator.

Two school systems participated in the study (Madison and Janesville, Wisconsin). The following results were common to both school systems:

(1) The ratio-denominator pictorial test was significantly easier than the fraction-denominator pictorial test for the low and middle ability levels in each grade.

(2) The fraction-numerator pictorial test was significantly easier than the fraction-denominator pictorial test for each ability group in each grade.

(3) The high ability children performed significantly better than the low ability children on all tests.

(4) The fifth and sixth graders performed significantly better than the fourth graders on all tests.

(5) Very low correlation existed between the scores on the symbolic test and the scores on the pictorial tests.

(6) The fraction-denominator test was the most difficult for each ability group in each grade.

VI. Report Title: Objectives of "Patterns in Arithmetic" and Evaluation of the Telecourse for Grades 1 and 3

Authors: James S. Braswell and Thomas A. Romberg

Date: January 1969

Summary:

Part I discusses the background and philosophy of "Patterns in Arithmetic" (PIA), with the mathematical objectives for grades 1-4 cited in detail.

Part II presents the results of an evaluation of PIA in grades 1 and 3 during 1966-67 for classes in Alabama and Wisconsin. Instruments used were both specially-developed and standardized achievement tests, and teacher and pupil inventories.

In Grade 1, the PIA group (in both states and separately) compared quite favorably with norms on both computation and concepts standardized tests. On the specially-developed tests, most objectives of the PIA program were found to be accomplished. Multivariate AOV analysis indicated that middle and high SES groups achieved more, especially on standardized tests.

In Grade 3, the PIA group also compared favorably with norms. On the pretest, groups in both states were slightly above the norm on concepts and below the norm on computation. On the posttest, the Alabama group was above the norm on both, while the Wisconsin group was better on concepts but lower than the norm on computation. On the specially-developed test, most PIA objectives were found to be accomplished.

One significant difference in achievement was found in favor of the Wisconsin group when the computation pretest was used as a covariate. When the concept test was used as a covariate, a significant difference favoring the high socioeconomic group was found.

Responses to the opinion inventories were generally favorable.

Project Title: Stanford Program in Computer-Assisted Instruction

Location: Palo Alto, California

Director: Patrick Suppes

Summary: The major activities of 1968 included:

(1) Brentwood Mathematics Program: 73 first graders were using the machines during the first part of the year. Modifications of the program were necessitated because of the slow machine response time. Analyses of data from the geometry are to be used for the second grade curriculum.

(2) Drill and Practice Math Program: Approximately 4,000, including teachers, aides, and students. The students were in elementary and junior high school, achieving on all levels. Use of this program and teacher workshops resulted in revisions in curriculum and development of automata for various programs.

(3) Logic and Algebra Program: students have completed the first year of the program, and begun the second year. The program for the third year has been completed.

(4) Dial-a-Drill Program: 20 students in grades 2-4 were involved. It was found that they preferred branching criteria after each problem in the strand.

Project Title: Stanford-Ravenwood Computer-Assisted Instruction Project

Location: Ravenwood, California

Director: William Rybensky, East Palo Alto

Purpose: This is the fourth revision to the original drill and practice materials developed by the Stanford Project, and this study is an evaluation of that revision in the eight schools of the Ravenwood School District

Procedure: A workshop was conducted to teach the manipulation and the programs for the Computer-Assisted Instruction. The program will operate with thirteen or fourteen "strands," including addition, division, fractions, etc. Each student can proceed at his own pace from grade one through junior high level material.

Results: Weekly conferences for evaluation are continuing.

Project Title: A Systems Approach to Improving Mathematics Education (SAM) (ES 001 711)

Location: Pittsburgh, Pennsylvania

Director: Frank Schilling, School District of Pittsburgh

Purpose: SAM attempts to improve the achievement of low-achieving fourth graders by providing teachers with a curriculum materials package.

Procedure: The system includes:

(1) forty-two sequential performance levels, with behavioral objectives
(2) forty films designed to introduce a performance level, motivate children and develop favorable attitudes toward mathematics
(3) discussion sheets for each film
(4) performance tests for diagnostic and evaluative purposes
(5) differentiated lesson plan sheets
(6) supplementary materials

The teacher shows a film, diagnoses with the performance tests the mastery of the behavioral objectives the film displayed, begins differentiated instruction using the instructional resources provided, evaluates the learning, and then chooses either to move on to the next behavioral objective or remain with the present one.

In-service work during the summers is supplemented by weekly meetings.

Results: Teachers indicated that (1) pupils seemed to show greater interest and enjoyment of mathematics, and (2) teachers were using extensive grouping and individualized instruction. They stated their preference for SAM over non-systematic instructional procedures.

Project Title: Use of Computer Assisted Instruction for Mathematics In-Service Education of Elementary School Teachers

Location: Williamsport, Pennsylvania

Directors: Samuel M. Long, Williamsport School District, and C. Alan Riedesel, The Pennsylvania State University

Purpose: The purposes are: (1) to study the feasibility of using computer-assisted instruction as a vehicle to teach in-service elementary teachers modern mathematics content; (2) to compare an in-service program offered by CAI with a program using conventional controls; (3) to provide an opportunity for demonstration to personnel from other school districts who may be interested in the future use of CAI; and (4) to improve the existing modern mathematics course by analyzing and using information from student performance.

Procedure: Four CAI student terminals were installed in a Willamsport elementary school. Seventy-eight teachers who had had no in-service work in modern mathematics were given a pre-test on concepts of modern mathematics. Teachers were placed in CAI and non-CAI groups on a stratified random basis. After a period of in-service instruction, posttests were administered and observations of the classroom behaviors of a random sample of teachers were conducted.

Results: Both treatment groups inproved in their knowledge of mathematical content. Teacher attitude toward CAI proved to be positive. There was great variation (14 to 47 hours) in the time required for teachers to complete the CAI program; thus demonstrating that CAI does treat individual differences. CAI can be considered a valid vehicle for improving the mathematical content of in-service teachers.

Additional information on the following projects was not yet available:

Title: Individualized Diagnostic Curriculum in Reading and Mathematics
Location: U.S. Dependents Schools (DOD), Europe
ERIC Number: ES 001 732

Title: Regional Elementary School Teacher Upgrading Project
Location: Watertown, Connecticut
ERIC Number: ES 001 887

Summaries of Additional ERIC-Documented Projects

In this section are included summaries of additional documents deposited in the Educational Resources Information Center (ERIC). All of these reports are listed in Research in Education, published by ERIC, and are available in either hard-copy or microfiche form.

The annotation for each report includes a brief statement of the most pertinent and valid results, findings, conclusions, or implications of the project. For more complete information, the report itself should of course be consulted.

Title:	Project for an Automated Primary-Grade Reading and Arithmetic Curriculum for Culturally Deprived Children
Investigators:	Richard C. Atkinson and Patrick Suppes
Dates:	1967, 1968
ERIC Numbers:	ED 014 506, ED 023 773

Summary or Findings:

Computer-assisted instruction was provided for culturally deprived children in the first grade. Material was designed not to require verbalization skills. Findings showed discipline problems decreased, and immature children acted more self-reliant. Children were so involved with the machine that the individual attention first graders need was fulfilled and the antics of the usual discipline-problem children were limited to activities not bothering others. Individualized instruction allowed each student to work on his own level, as well as at his own rate. Success was rewarded with a picture of a happy face so that children on all levels could feel satisfaction. The drawback of this program was the amount of breakdown time of the computer.

Title: Comparison of Two Teaching Techniques in Elementary School Mathematics

Investigator: Otto C. Bassler

ERIC Number: ED 023 595

Summary or Findings:

Groups taught operations with integers, modulus arithmetic or vector arithmetic achieved most when texts used a linear program format with a high level of guidance.

Title: Organization of Mathematics in Grades 4, 5 and 6

Investigators: Kathryn A. Blake and others

Date: 1966

ERIC Number: ED 010 039

Summary or Findings:

Mathematical content in grades 4, 5 and 6 was organized in vertical and horizontal sequences. A vertical hierarchy of complexity was present among the subtopics with the topics analyzed. The following sequences were found within each topic:

(1) Number operations: (a) addition, (b) subtraction, (c) multiplication, (d) division, (3) laws.

(2) Geometry: (a) definition of terms, (b) recognition, (c) measurement.

(3) Relations: (a) maps, (b) graphs, (c) charts, (d) equality, (e) function.

(4) Numeration: (a) reading, (b) writing, (c) use, (d) bases, (e) sets.

(5) Measurement: (a) concept of measurement, (b) non-conversion, (c) conversion, (d) operation.

(6) Fractions (sixth graders only): (a) concepts, (b) addition, (c) subtraction, (d) multiplication, (e) division.

Each separate concept was horizontally sequenced, orthogonally and non-orthogonally. The empirical orthogonal hierarchy was measurement, geometry, numeration, and whole numbers. The non-orthogonal hierarchy was whole numbers, numeration, geometry, fractions and relations.

Title: Evaluation of the Madison Project Method of Teaching in Arithmetic Situations, Grades 4, 5, and 6.

Investigator: William F. Brown

Date: 1963

ERIC Number: ED 003 053

Summary or Findings:

The achievement and attitude of fourth, fifth, and sixth grade students, learning by the Madison Project method of teaching using Socratic questioning, was compared to students learning by the traditional method taught in Syracuse, New York. Achievement test results showed the experimental group achieved more than the control group in the fourth grade. Results were very close in fifth and sixth grade, with the experimental group doing slightly better in the fifth grade, and the control group doing slightly better in the sixth grade.

On an attitude scale, the control group appeared to have a more positive attitude towards mathematics.

Title: Arithmetical Abstractions--The Movement Toward Conceptual Maturity Under Differing Systems of Instruction

Investigator: William A. Brownell

Date: 1964

ERIC Number: ED 003 299

Summary or Findings:

English and Scottish schools participated in a study concerning the teaching programs of mathematics concepts. Emphasis was on programs which enhanced conceptual maturity among children. Relative effectiveness of one program over another was not found.

Title: Arithmetical Computation: Competence After Three Years of Learning Under Differing Instructional Programs

Investigators: William A. Brownell and others

ERIC Number: ED 022 703

Summary or Findings:

Low I.Q. pupils appeared to gain more from use of the Cuisenaire program than from a traditional program in Scotland.

Title: An Experimental Study of the Influence of Individual vs. Group Instruction on Spatial Abilities in Preschool Children

Investigator: Sister J. Concannon

Date: 1966

ERIC Number: ED 010 126

Summary or Findings:

Children attending Montessori and non-Montessori classes were compared as to haptic perception. The Montessori-trained children performed significantly better in haptic perception tasks. There were no significant differences in performance between the sexes, those attending morning and afternoon sessions, those with previous and no previous schooling, differences in mental age, or those having individual and group instruction.

There was a significant difference in performance among those with differences in chronological age.

Title: Research and Development Activities in R and I Units of Two Elementary Schools of Janesville, Wisconsin, 1966-67

Investigators: Doris M. Cook and others

Date: 1968

ERIC Number: ED 023 175

Summary or Findings:

Teacher demonstration followed by pupil manipulation of interesting concrete objects was most effective.

Title: Mathematics Teaching Behavior Changes Made by Intermediate Grade Teachers During a 15-Week Period of Instruction by Educational Television

Investigator: Clyde G. Corle

Date: 1967

ERIC Number: ED 015 657

Summary or Findings:

A forty-five week in-service educational T.V. course for fourth, fifth, and sixth grade mathematics teachers was the basis for quantifying behavior changes. Sixteen pairs of teachers were chosen from six districts. One teacher from each pair participated in the television course, while the other teacher of the pair was not allowed to watch it.

Rating on eight scales was done by seven trained observers on a rotating basis seven times prior to the television course and twenty-three times during the television course. Scores were analyzed by a covariance technique.

On two of the scales, there were differences between the strategies of experimental and control teachers. The results of achievement tests showed the gain score significantly higher among the experimental teachers than among the control teachers.

The author of the study felt that fifteen weeks was too short a time to document behavior changes, and that the twenty-minute observation period did not provide enough teacher-pupil interaction to justify use of the observation instrument.

Title: The Reading-Arithmetic Skills Program, A Research Project in Reading and Arithmetic

Investigators: Clyde G. Corle and Myron L. Coulter

Date: 1964

ERIC Number: ED 010 989

Summary or Findings:

Students were given additional reading help for verbal problems. The mean gains in the arithmetic achievement test, scores, and gains within subtests were positive. The size of correlations within subtests increased as the grade level increased.

Title: The Development of an Information System for Elementary School Mathematics Curriculum Materials --A Feasibility Study

Investigators: Doris E. Creswell and Emanuel Berger

Date: 1967

ERIC Number: ED 015 768

Summary or Findings:

Objectives of PRIMES, which includes a microfilm document file of 25,000 analyzed pages from elementary mathematics basal texts, are explained.

Title: A Modern Mathematics Program as it Pertains to the Interrelationship of Mathematical Content, Teaching Methods and Classroom Atmosphere (Madison Project)

Investigator: Robert B. Davis

Dates: 1963, 1965, 1967

ERIC Numbers: ED 003 369, ED 003 371, ED 017 458

Summary of Findings:

The areas in elementary education into which the Madison Project extends are presented. In the area of teaching methodology, experiences in which the student has not previously participated are provided. Exploratory experiences are emphasized. The philosophy of the Madison Project is to have students actively participate in a diversity of types of activities and to be aware of a variety of mathematical concepts. The proposed curriculum includes an earlier exposure of multiplication, division, geometry, algebra, and other advanced mathematics.

Teacher preparatory and in-service projects include increasing the repetoire of teachers and school administrators. Aids such as descriptions of exploratory experiences, tape recordings of actual classroom activities, and films are incorporated into the teacher education programs. New concepts in content and learning theory are presented to publishers; thus they can be made available in tests.

Title: Effects of a Structured Program of Preschool Mathematics on Cognitive Behavior

Investigator: Therry N. Deal

ERIC Number: ED 015 791

Summary or Findings:

Seventy pre-school children participated in a study testing the effectiveness of teaching one-to-one correspondence and the child's concept of conservation. Three experimental classes were contrasted to three control classes of a general pre-school program to facilitate mathematical understanding. A pretest showed the two groups were not significantly different.

The posttests showed the experimental group performed significantly better in tests of one-to-one correspondence, but there was no significant difference in their concepts of conservation.

The older pre-school children performed significantly better in tests of concepts of conservation than did the younger pre-school children. However, the increase in mean scores was approximately equal for each age group.

Title: The Effect of an Individually Prescribed Instruction Program in Arithmetic on Pupils at Different Ability Levels

Investigator: Donald Deep

Date: 1966

ERIC Number: ED 010 210

Summary or Findings:

An IPI (Individually Prescribed Instruction) experimental fourth, fifth, and sixth grade group was compared to a sixty-six student matched group, as a control. Using IPI, the higher ability students tended to do more work than the average low group.

The results of the standardized tests showed no significant difference among high, average, and low ability groups in computation and problem solving when the pretest is taken into account.

Title: Television and Consultant Services as Methods of In-Service Education for Elementary School Teachers of Mathematics

Investigators: Vere M. DeVault and others

Date: 1962

ERIC Number: ED 003 562

Summary or Findings:

The relative effect of television or face-to-face lecture-discussion for in-service training was compared. Classroom consultant services, averaging 5 visits per teacher with a total average of 5 hours per teacher, were supplied to one-half of each group. Findings were:

(1) Television instruction was effective as face-to-face lecture discussions, with significant achievement gains being made by both groups of teachers.

(2) Consultant services made a significant contribution to teachers' and pupils' achievement.

(3) Pupils in heterogeneous groups made significant gains in both treatments in terms of mathematics achievement related to the in-service program but not on the STEP measures.

It was concluded that the consultant services were significant contributing factors for effective in-service education of teachers.

Title: Psychological Problems and Research Methods in Mathematics Training

Investigators: Philip H. DuBois and Rosalind L. Feierabend

Date: 1959

ERIC Number: ED 002 915

Summary or Findings:

A handbook was developed to: (1) formulate problems in the learning of mathematics which might be answered through appropriate experimental investigations, (2) suggest research methods and appropriate designs, and (3) review psychological and educational literature for 1948 to 1958, reporting all relevant research. The three parts of the handbook are:

I. "Problem Areas in Teaching Mathematics," papers presented at a conference on psychological problems and research methods in mathematics training held at Washington University in 1959.

II. "Proposals for Research on Teaching Mathematics" also by above participating speakers.

III. "A Review of Research in Mathematics Education" for 1948 to 1958 dealing with comparisons of teaching methods, relation of proficiency in mathematics to thought process, motivation, mental abilities, personality and evaluation and measurement, including foreign research.

Title: Exploratory Type of Evaluation of TV Training of Elementary Mathematics Teachers

Investigators: Leslie A. Dwight and others

Date: 1966

ERIC Number: ED 011 057

Summary or Findings:

Classroom and television instruction for in-service teachers was studied. An eight-test battery was the instrument for the pretest and posttest. Both groups showed achievement after instruction. Those teachers having the classroom instruction did better on each subtest of the battery than those having the television instruction. The achievement difference between groups increased with time.

Title: Study of Mathematics Teachers in Alabama

Investigator: Kenneth Easterday

Date: 1967

ERIC Number: ED 014 454

Summary or Findings:

Questionnaires returned by 964 secondary and elementary teachers in Alabama indicated:

(1) 62.5% had 4-year certificates, 26.9% had 5-year certificates, and 3.4% had 6-year certificates.

(2) Median experience was 9.4 years; 20% were new to the current school.

(3) 46.3% of the secondary teachers had not majored in mathematics, while 32.4% with a mathematics major taught other courses. 39.6% had attended National Science Foundations programs.

(4) Average class size was 25 for secondary courses (all traditional) and 35 in elementary schools.

(5) Of the CUPM recommendations, less than 10% of elementary teachers and 25% of the junior high teachers reported equivalent course work; 25% of secondary teachers reached level 3 requirements.

Conclusions were that there is a need for re-training programs for upper elementary and junior high teachers, plus an increase in salaries.

Title: Self-Selected Mathematics Learning Activities
Investigator: William M. Fitzgerald
Date: 1965
ERIC Number: ED 003 348
Summary or Findings:

Seventh and eighth graders participated in an individual, self-selection mathematics curriculum program. Achievement, attitude, age, and the qualities of industriousness and activeness were examined.

It was shown that the bright students (115 I.Q. and over) did not learn as much in the self-selection classes as did those in the conventional classes. Slower students (114 I.Q. and below) learned equally well in both classes.

The students in the experimental classes became more industrious and participated in group situations more often as time went on. The girls were found to be more industrious but less active, and they worked by themselves more than the boys did. The brighter students as a group were more industrious than the slower students.

Title: Foundations of Mathematics for Elementary Teachers
Investigator: E. Glenadine Gibb
Date: 1966
ERIC Numbers: ED 011 517, ED 024 658
Summary or Findings:

Video tapes, a consultant's guide, and a textbook on foundations of mathematics were prepared, as an overview for teachers.

Title: Sequentially Scaled Mathematics Achievement Tests --Construction, Methodology and Evaluation Procedures

Investigator: Glenn T. Graham

Date: 1966

ERIC Number: ED 010 211

Summary or Findings:

Methodology for test construction was examined, and evaluation procedures for reliability, validity, and item analysis for a test were ascertained. Concepts of addition, subtraction, numeration, time-telling, and money in grades one to three were included on the test.

It was found to be possible to construct sequentially scaled achievement tests in certain areas of arithmetic. Guttman's scalogram analysis methodology was applied to the test construction.

Title: The Effects of Selected Experiences on the Ability of Kindergarten and First-Grade Children to Conserve Numerousness

Investigators: E. Harold Harper and Leslie P. Steffe

Date: 1968

ERIC Number: ED 021 752

Summary or Findings:

Lessons on such concepts as one-to-one correspondence and relative quantity helped kindergarteners gain in ability to conserve.

Title: Maryland Elementary Mathematics In-Service Program

Investigators: James Henkelman and others

Date: 1967

ERIC Number: ED 012 397

Summary or Findings:

The developmental phase of an in-service program was described. The tasks carried out were:

(1) Staff construction of behavioral objectives
(2) Development of a behavior action-verb list
(3) Constructing a behavioral hierarchy
(4) Identification of terminal tasks with the highest yield for in-service education

Presentation and explanation of algorithms with the behaviors to be exhibited at termination was chosen for a behavioral hierarchy. Instructional materials were developed for the hierarchy, revised, and used with two classes (28 teachers) for in-service education in a pilot study. Over 50% of the teachers acquired the terminal behaviors; further revision of materials was indicated.

Title: Development and Evaluation of Auto-Instructional Programs in Arithmetic for the Educable Mentally Handicapped
Investigators: Conwell Higgins and Reuben R. Rusch
Date: 1967
ERIC Number ED 014 190
Summary or Findings:

Educable mentally retarded children used audio-visual manipulative devices for learning concepts of addition, subtraction, more than, less than, and ordinals, for the numbers one through five.

Visual information was presented using a slide projector, and audio-information was heard from either earphones or loud speakers. The child responded by either moving objects or by writing on his response desk. Each child worked each program twice.

This device was found to be effective in a classroom setting under the supervision of a classroom teacher. It was not suggested for trainable mentally retarded children.

Title: A Half Century of Teaching Science and Mathematics

Investigators: Katherine U. Isenbarger and others

Date: 1950

ERIC Number: ED 011 839

Summary or Findings:

A fifty-year historical review of mathematics in the schools and for teacher preparation is given. Included are changes in courses, placement, teachers associations and publications, important committees and commissions, function of mathematics in education, and curriculum trends. The history of teacher preparation includes early courses, curriculums, degree of preparation, factors influencing change, legal requirements, influential reports, present status, and trends and needed emphasis.

Title: Class Size and Achievement Gains in Seventh- and Eighth-Grade English and Mathematics

Investigators: Mauritz Johnson and Eldon Scriven

Date: 1967

ERIC Number: ED 016 653

Summary or Findings:

The effects of homogeneous and heterogeneous grouping and class size was the basis of this study with seventh and eighth grade mathematics students. Classes of twenty-nine students were considered small; over 29, large. Arithmetic test scores on the Iowa Test of Basic Skills showed gains which were small, inconsistent, and not statistically significant even between groups with extreme differences in size.

Title: Use of the Semantic Differential Technique to Measure Prospective Elementary School Teacher Attitude Toward Mathematics and Other Subjects

Investigator: Robert B. Kane

ERIC Number: ED 021 761

Summary or Findings:

The study reports on comparative attitudes as well as on the development of the scales.

Title: Abilities of First-Grade Pupils to Learn Materials in Terms of Algebraic Structures by Teaching Machines

Investigators: Evan R. Keisler and Robert C. Crawford

Date: 1961

ERIC Number: ED 003 038

Summary or Findings:

The abilities of first grade pupils to learn materials in terms of algebraic structures by a teaching machine were contrasted with the abilities of first and fourth graders to learn algebraic structures in a conventional classroom situation.

The experimental group learned order of numbers without loss in addition and subtraction competencies. Results were not given, but it was stated that the results hinged on the judgment of the validity of the test as a major criterion.

Title: Bridging the Grade Six to Seven Gap With Continuous Progress

Investigator: Jeremiah J. Kellett

Date: 1966

ERIC Number: ED 010 109

Summary or Findings:

Seventh and eighth grade level self-study mathematics materials were given to sixth and seventh grade students. The attitude of these students towards mathematics, and their mathematical achievement, showed no significant change.

Title: An Analysis of Learning Efficiency in Arithmetic of Mentally Retarded Children in Comparison With Children of Average and High Intelligence

Investigators: Herbert J. Klausmeier and others

Date: 1959

ERIC Number: ED 002 781

Summary or Findings:

Acquisition and retention of arithmetic tasks, as well as physical, emotional and achievement factors, were investigated for high, average, and low intelligent children, ages 103 to 125 months of age. Arithmetic tasks related to counting, addition, subtraction, common problem solving, and differential problem solving. Physical characteristics measured were height, weight, strength of grip, dentition, and carpel age. Emotional characteristics investigated were peer acceptance, expression of emotion, behavior patterns, adjustment, and self-concept. Analysis of variance, analysis of covariance and correlational techniques were used. The major conclusions were:

(1) The ratio between learning-relearning time was the same among children of low, average, and high I.Q., but rate of acquiring was longer for low I.Q.'s.

(2) Retention of arithmetic learning was the same for all groups (when the original learning task was appropriately graded to the achievement level of the child) for counting, subtraction, addition, and differential problem solving.

(3) Uneven physical development within the child did not accompany low efficiency in arithmetic and reading achievement. A trend was apparent but not significant.

(4) Low level of physical development within the child did not accompany low achievement in reading and arithmetic except for boys at the mean age of 125 months.

(5) The low and high I.Q. children showed more variability than the average in I.Q., reading, arithmetic, language achievement, and strength of grip.

(6) Emotional adjustment, achievement in relation to capacity, and integration of self-concept correlated positively and generally significantly for both sexes of all groups.

(7) Expression of emotion, behavior patterns, and the child's estimate of his own abilities showed positive and usually significant correlations for both sexes at 113 and 125 months of age.

(8) Level of physical development would seem quite useless in attempting to understand a child's pattern of failures and success in most school activities.

Title: Concept Learning and Problem Solving--A Bibliography

Investigators: Herbert J. Klausmeier and others

Date: 1965

ERIC Number: ED 010 201

Summary or Findings:

A variety of definitions of "concept" and taxonomy of variables significant in concept learning are presented. A bibliography of articles from 1950-1964 concerning concept learning and problem solving is given.

Title: Feasibility of a New Basis for School Arithmetic

Investigator: George Klein

Date: 1968

ERIC Number: ED 021 753

Summary or Findings:

Common mechanical devices were shown to be effective in upgrading achievement at the fourth grade level.

Title: Investigations on the Feasibility of Some Extensions of School Arithmetic

Investigator: George Klein

ERIC Number: ED 024 572

Summary or Findings:

Infinite decimal arithmetic can be built on or developed independently of fractions.

Title: The Development of an Elementary School Mathematics Curriculum for Individualized Instruction

Investigators: Joseph I. Lipson and others

Date: 1966

ERIC Number: ED 013 120

Summary or Findings:

The philosophy and organization of the IPI curriculum are discussed. Achievement results for 1964-65 showed a wide range.

Title: Report of the International Clearinghouse on Science and Mathematics Curricular Developments

Investigators: J. David Lockard and others

Dates: 1967, 1968

ERIC Numbers: ED 014 438, ED 020 143

Summary or Findings:

Reports on current project groups in over forty countries are listed.

Title: Mathematics Education Programs Funded Under Title I Elementary and Secondary Education Act of 1965

Investigator: Melvin Mendelsohn

Date: 1968

ERIC Number: ED 017 459

Summary or Findings:

This is one of a series of booklets describing 25 continuing Title I projects.

Title: The Effect of Various Training Techniques on the Acquisition of the Concept of Conservation of Substance

Investigators: Egon Mermelstein and others

Date: 1967

ERIC Number: ED 014 433

Summary or Findings:

Kindergarten children were subjected to training techniques to teach the Piagetian concept of conservation of substance. The four training methods were: (1) cognitive conflict (one-to-one correspondence established, then confused); (2) multiple classification (several names and properties given to a single item followed by reversing of training); (3) verbal explanation (experimenter's reasons for occurrences given orally are combined with multiple classification); and (4) language activation (a verbal expression of occurrences by the child).

The two non-verbal tasks in which the law of conservation of substance was violated, and the one Piagetian test of conservation of substance, showed that the concept of conservation of substance was not induced by any of the training techniques employed, that language interferes with acquisition of this concept, and that problems of reversals merits further exploration.

Title: Research Memorandum: Evaluation of an Inservice Television Training Program in Mathematics for Elementary Teachers

Investigators: Donald Mills and others

Date: 1965

ERIC Number: ED 003 162

Summary or Findings:

Teachers viewed fifteen 30-minute telecasts, participated in discussions, and used guide books for reading and working examples concerning modern mathematics. Pre- and posttesting for attitudes and achievement were administered. Only 54% (192 teachers) of the total sample provided usable data. Because of ambiguous results, conclusions reached were that (1) teachers did not benefit from the instruction, or (2) the measurement instruction used was not appropriate.

Title: Arithmetic Concepts of Kindergarten Children in Contrasting Socioeconomic Areas

Investigator: David Montague

ERIC Number: ED 014 503

Summary or Findings:

What children from different SES levels know when they enter school is enumerated.

Title: A Study of Development of Conservation of Quantity

Investigators: Herbert H. Muktarian and George G. Thompson

Date: 1966

ERIC Number: ED 010 261

Summary or Findings:

One group of five and six year olds, given a complex task typical of Piaget, was compared to an experimental group given another task which was a measure of conservation of quantity that is independent of Piaget's theoretical formulation. The experimental group focused on logical permanence prior to being tested on the same test as the control group.

The experimental group was able to conserve quantity when given the proper sequence. Stability of performance in conserving indicated that the operation was meaningful to them. This implies that kindergarten children, given skillful guidance, can work with quantity as a meaningful concept.

Title: The Formation of Addition and Subtraction Concepts by Pupils in Grades One and Two

Investigator: Joseph N. Payne

Date: 1967

ERIC Number: ED 015 015

Summary or Findings:

The part-part-whole contrasted to the take-away approach of teaching subtraction, and teaching symbolization early contrasted to teaching symbolization later, were the two basic concepts tested with first and second grade classes.

The only advantage of the take-away method was its being easier to teach. The part-part-whole method was understood by both first and second graders. After grade two, the part-part-whole method was considerably better for addition and subtraction for transfer of learning, and for relating addition and subtraction.

Those children introduced to symbolization early achieved at a higher level than those who were introduced to it later.

There was no difference in achievement between the sexes, nor was there any difference in attitude towards mathematics in any of the groups.

Title:	A Comparison of Automated Teaching Programs With Conventional Teaching Methods as Applied to Teaching Mentally Retarded Students
Investigator:	James E. Price
Date:	1961
ERIC Number:	ED 003 654

Summary or Findings:

Efficiency of the mentally retarded students learning addition and subtraction the conventional way or using teaching machines with multiple-choice programmed learning and answer-construct programmed learning were compared. All three groups improved with instruction in addition. Only the multiple-choice programmed learning group improved significantly with instruction in subtraction.

Title:	A Study of Psychological Patterns in Learning Elementary Mathematics
Investigator:	Frank Restle
Date:	1966
ERIC Number:	ED 010 184

Summary or Findings:

Sequences in counting a group of objects, methods for solving a problem, adding large numbers involving carrying, and multiplying were investigated.

The construction of a variety of tasks emphasized the slightly different discriminations.

Title: Teaching Mathematics Through the Use of a Time Shared Computer

Investigator: Jesse O. Richardson

Date: 1968

ERIC Number: ED 023 606

Summary or Findings:

Programming languages were effectively taught to elementary pupils.

Title: The Use of Games to Facilitate the Learning of Basic Number Concepts in Preschool Educable Mentally Retarded Children

Investigator: Dorothea Ross

Date: 1967

ERIC Number: ED 023 243

Summary or Findings:

Games were found effective; various techniques for their use are suggested.

Title: Teaching of Advanced Mathematical Concepts to Culturally Disadvantaged Elementary School Children

Investigator: William H. Rupley

Date: 1966

ERIC Number: ED 011 081

Summary or Findings:

Description of a mathematician's experiences teaching culturally deprived second, third, fourth and sixth graders non-standard and advanced topics of mathematics by the discovery technique is presented.

Title: Reading Problems in Mathematics Texts

Investigator: Judith A. Shaw

Date: 1967

ERIC Number: ED 016 587

Summary or Findings:

California state-adopted textbooks for grades 1-8 were analyzed for readability levels, amount of expository and story problem reading, and frequency of mathematics vocabulary. Chapters were randomly chosen and word lists compared with Thorndike for vocabulary load. Findings indicated: (1) the greatest number of words were introduced at grade 4; (2) there was great internal variation of reading level in all texts; (3) there was a significant increase in expository and story problem reading in the 4th grade; (4) vocabulary load was lowest at primary and greatest at junior high; and (5) reading levels of the 7th grade texts showed the following discrepencies: (a) high-ability texts had 5th to 6th grade reading level, (b) low-ability texts had 7th grade reading level, and (c) middle-ability texts had 9th to 10th grade reading level.

Title: The Problems of Under Achievement and Low Achievement in Mathematics Education

Investigators: Dwain E. Small and others

Date: 1966

ERIC Number: ED 010 535

Summary or Findings:

Differences between twelve under-achievers and eleven low-achievers in grades four, five, and six, and their ability to function on the concrete, semi-concrete, and abstract levels, were studied using a clinical approach. Ability to function on the difficulty levels was found not to be related to low or under-achievement and no consistent pattern was evident among these two groups.

Psychological and sociological data from observations, teacher and parent interviews, cumulative records, California Test of Personality, and WISC, showed the under-achievers tended to come from homes which emphasized high achievement, and where excessive stress was on school grades and college preparation. The performance of these children was lowered considerably in high anxiety situations. The low-achievers tended to come from homes with poor family relationships, had personal and social adjustment problems, had poor behavior in school, and overall academic development was below par.

Title: Understanding of Concepts of Probability Theory by Junior High School Children

Investigators: Charles D. Smock and Gretchen Belovicz

Date: 1968

ERIC Number: ED 020 147

Summary or Findings:

Results indicated that students failed to understand the basic idea of probability theory.

Title: Elementary Arithmetic and Learning Aids

Investigator: Patricia M. Spross

Date: 1965

ERIC Number: ED 017 457

Summary or Findings:

This is a manual designed to help teachers identify and use appropriate manipulative and audio-visual aids.

Title: The Effects of Two Variables on the Problem-Solving Abilities of First-Grade Children

Investigator: Leslie P. Steffe

Date: 1967

ERIC Number: ED 019 113

Summary or Findings:

Problems with and without quantifiers did not cause achievement differences, but keeping names of sets consistent was helpful.

Title: The Development of the Concepts of Ratio and Fraction in the Fourth, Fifth and Sixth Years of the Elementary School

Investigators: Leslie P. Steffe and Robert B. Parr

Date: 1968

ERIC Number: ED 023 612

Summary or Findings:

Tests using fractions with missing denominators were more difficult than those with ratios or missing numerators.

Title: Comparative Studies of Principles for Programming Mathematics in Automated Instruction

Investigators: Laurence M. Stolurow and Max Beberman

Date: 1964

ERIC Number: ED 013 377

Summary or Findings:

The basic principles for automated instruction using linear and branching design were the basis for the study. There was no achievement difference between the students using these types of instruction after seven weeks, except that high ability students, regardless of group, had higher scores. The branch design was more efficient but the linear design was preferred by the students.

Title: Stanford Program in Computer-Assisted Instruction

Investigators: Patrick Suppes and Richard C. Atkinson

Dates: 1967, 1968

ERIC Numbers: ED 016 632, ED 016 633, ED 021 441

Summary or Findings:

These are three of the progress reports, which are prepared quarterly.

Title: Experimental Teaching of Mathematical Logic in the Elementary School

Investigators: Patrick Suppes and Frederick Binford

Date: 1964

ERIC Number: ED 003 367

Summary or Findings:

The achievement of five hundred fifth and sixth graders exposed to mathematical logic was compared to that of college undergraduate and graduate students. The upper quartile in intelligence (I.Q.'s of 142-184) of fifth and sixth grade students achieved 85% to 90% of that achieved by the college students. The actual hours of instruction were approximately equal for both groups. However, the fifth and sixth grade classes had more short time segments, compared to a more compact period of instruction for the college students.

Suggested teacher training was a five- to six-semester-hour course.

Transfer to arithmetic, reading, and English was greatest among the brightest students. The advanced summer course group showed rapid progress in other phases of mathematical logic.

Title: Arithmetic Drills and Review on a Computer-Based Teletype

Investigators: Patrick Suppes and others

Date: 1965

ERIC Number: ED 014 215

Summary or Findings:

A computer terminal was used by 41 fourth graders (average I.Q. 130) for drills in division and multiplication in combination with addition and subtraction. Examples were dispersed and not sequenced as to type. Information on the average proportion of errors, successes, and time-outs (to differentiate from errors) was available for teacher planning. The main problems of the program concerned timing procedures (students needing extra time or unnecessary time between students).

No statistical evidence of the success of the computer drill over other types of drill was available.

Improvements suggested were more terminals, different programs for students with different achievement on the previous lesson, and a more detailed report to students and teacher.

Title: Individualizing Junior High School Mathematics Instruction

Investigator: Joseph T. Sutton

Date: 1967

ERIC Number: ED 016 609

Summary or Findings:

A seventh grade class using an individualized mathematics unit with exercises and tests was compared to a control class. This experiment took place in small schools which were created to relieve scheduling difficulties in the larger schools. Results, as measured by the California Achievement Test, showed the control classes significantly higher than the experimental in reasoning and fundamentals.

Title: Compilation of Research Results in Elementary Arithmetic Since 1900

Investigator: Marilyn N. Suydam

Date: 1967

ERIC Number: ED 013 227

Summary or Findings:

This study was designed to meet the need for a basic source of information on the research in elementary school mathematics. A summary of the type of information presented in some detail in the study includes:

1. A list of all reports of research which relate to the teaching of mathematics in the elementary school and which have been printed in journals published in the United States during the years from 1900 through 1965 was compiled. A total of 799 research reports were found in 50 journals.

2. Each study is categorized by mathematical topic and type of study. Of the total, 207 were placed in the categories for educational objectives and instructional procedures; 63 in topical placement; 154 in basic concepts and methods of teaching them; 78 in materials; 131 in individual differences; 99 in evaluating progress; and 67 were categorized as studies related to learning theory. The frequency by types of studies was: descriptive, 107; survey, 230; case study, 18; action, 63; correlational, 56; ex post facto, 79; and experimental, 246.

3. The research which is experimental is also categorized by design paradigm. Of the 246 experimental studies, 39 involved no control groupl 150 involved possible sampling errors; while only 57 seemed to be valid examples of more carefully designed experiments.

4. Specific information on statistical procedure, variables controlled, sampling procedures and size, type of test, grade level, and duration are included whenever applicable in the analysis of each report.

5. Major conclusions which appear consistent with the data in each study are also noted with the analysis of each report.

6. An Instrument for Evaluating Experimental Research Reports was developed and tested for reliability. In one study with three judges, the interrater agreement was found to be .91, while the intraclass reliability was .77. In a second study with twelve judges, the interrater reliability was found to be .94, with an intraclass reliability coefficient of .58.

7. The experimental research is evaluated with this instrument. None of the reports was rated excellent in overall rating. Thirty-four of the reports were rated very good; 60, good; 84, fair; and 68, poor.

8. Each study was assigned to a composite evaluative category. The non-experimental group included 553 reports; 112 seemed sound and pertinent to curriculum today under the stated definition of experimental research; for 9, the purpose did not seem pertinent; for the remaining 125 the purpose seemed pertinent to curriculum today, but type of study, design, and/or statistical procedures did not seem sound and/or accurate today under the stated definition of experimental research.

9. A list of dissertations which were completed through 1965 was compiled and included in an appendix to increase the comprehensiveness of the compilation.

10. Pertinent data were summarized and major conclusions pertaining to mathematical and educational research methodology are listed. No clearly defined applicability to a theory of instruction is evident.

Title: Characteristics of Mathematics Teachers That Affect Students' Learning

Investigators: E. Paul Torrance and others

Date: 1966

ERIC Number: ED 010 378

Summary or Findings:

In this study the effectiveness of one hundred twenty-seven mathematics teachers in grades 6 to 12 was related to teacher-pupil interaction, classroom climate, creative abilities of the teachers, and conventional qualifications of teachers. Teachers participating used experimental SMSG instructional materials.

Pre- and posttests measured the students' attitude, aptitude, and educational and mathematical progress.

It was found that if teachers met minimum qualifications, higher qualifications in the following areas made no difference: (1) length of experience, (2) undergraduate and graduate courses and grades, and (3) participation in mathematics organizations.

The most effective teacher had a greater variety of ideas indicative of successes and failures in their teaching. They hypothesized causes of their success and failures and produced a greater variety of alternative ways of teaching.

[See also: Paul C. Rosenbloom and others. Characteristics of Mathematics Teachers That Affect Students' Learning (ED 021 707). 1966.]

Title: Training Effects and Concept Development--A Study of the Conservation of Continuous Quantity in Children

Investigator: J. O. Towler

Date: 1968

ERIC Number: ED 016 533

Summary or Findings:

Students were pretested to determine whether each was a conserver, a partial conserver, or a non-conserver. Students discussed their answers and did other similar problems. The final testing showed that the non-conservers become conservers at a significant level. Non-conservers becoming partial conservers and conservers regressing were not significant.

Title: Problem-Solving Proficiency Among Elementary School Teachers

Investigator: Richard Turner

Date: 1960

ERIC Number: ED 002 974

Summary or Findings:

The relationships between variables and ability to solve teaching problems were investigated with 154 teachers, graduate and undergraduate students.

Measures of values (Vernon-Lindsay) and attitudes or dominant interests were not relevant to problem solving abilities. Opportunities to learn (exposure to methods courses and student teaching) were significantly relevant to increased performance of problem solving abilities.

Teachers of kindergarten through grade 3 did not perform as well as grade 4-6 teachers. Performance varied with type of experience and preparatory institution, but not significantly.

Title: Problem Solving Proficiency Among Elementary School Teachers

Investigator: Richard L. Turner

Date: 1961

ERIC Number: ED 011 045

Summary or Findings:

The reliability and validity of seven problems in teaching arithmetic were investigated with 136 teachers, 41 non-teachers and 195 preparatory teachers as subjects. The problems were concerned with (1) examples of worked problems for subjects to analyze errors and judge actions to be taken, (2) judgment of exercises in relation to objectives, (3) ordering problems for difficulty, and (4) stated meanings of subtraction and division. Conclusions were:

(1) Teacher group had the best overall performance.

(2) Maximum performance was in the early years of experience.

(3) Age up to 50, study of value scores, grade level taught, recency of methods course, number of mathematics courses, sex and size of preparatory institution were not significant factors.

(4) Size of institution, A.C.E. scores and reading comprehension were significant in relation to variables and to each other.

Title: Review of New Mathematics Curriculum Materials

Investigator: Carl J. Vanderlin, Jr.

Date: 1966

ERIC Number: ED 022 665

Summary or Findings:

A description of 16 curriculum projects and materials available from them are included.

Title: Research and Development Activities in R and I Units of Two Elementary Schools of Manitowoc, Wisconsin, 1966-1967

Investigators: James L. Wardrop and others

Date: 1967

ERIC Number: ED 019 796

Summary or Findings:

Pupils with average ability and achievement performed better in homogeneous groups, while low achievers were better in heterogeneous groups and both groups were satisfactory for high achievers.

Title: Intellectual Processes Related to Mathematics Achievement at Grade Levels 4, 5, and 6

Investigators: Helen R. Westbrook and others

Date: 1965

ERIC Number: ED 003 669

Summary or Findings:

The relationships between certain intelligence components and mathematic achievement topics were investigated for 4th through 6th graders. Intelligence components considered were (1) perceptual discrimination, (2) memory span, (3) associative memory, (4) associative memory plus conversion, (5) reasoning, (6) verbal meaning, and (7) special relationships. Mathematics components measures were (1) numerical operation and assumptions, (2) geometry, (3) relations, (4) numeration, (5) measurement, and (6) operating with fractions.

The predominent psychological components of mathematics achievement were facility in reasoning (number relations) and facility in discerning verbal meanings (synomyms). These two contributed to the majority of mathematics topics and subtopics at all grade levels.

Title: Conservation, Cardination, and Counting as Factors in Mathematics Achievement
Investigator: Greyson H. Wheatly
SRIS Number: 0871 (Phi Delta Kappa)
Summary or Findings:

This study developed a test of conservation and counting. Conservation was found to be a significant predictor of achievement, while counting was not.

Title: Measures of Learning Rates for Elementary School Students in Mathematics and Reading Under a Program of Individually Prescribed Instruction
Investigator: John L. Yeager
Date: 1966
ERIC Number: ED 010 209
Summary or Findings:

Students in grades one through six were given individualized mathematics instruction. Their achievement scores depended upon the number of units mastered. No generalization could be applied to the results of the tests, since the result depended upon the individual units completed.